PERCEIVING TRANSPARENCY

by Fabio Metelli

1. Physical and perceptual transparency

What does "transparent" mean? There are two different meanings of this word, which are often confused. When we say that a thing or a medium is transparent we may intend to say either that it is permeable to the luminous flux, or that we see things through it. The first is the physical, and the second the perceptual meaning of "transparent".

Of course, the distinction would not be very important if physical and perceptual transparency were always linked together, so that physical transparency always accompanied perceptual transparency and vice-versa. But this is not the case. Air is physically transparent, but normally (where there is no fog) one would not speak of "seeing through" it. Nor do we always perceive plate glass doors, since we often run into them. Keeping these examples in mind, it seems useful to give a more precise definition of the perception of transparency: one perceives transparency when he perceives not only surfaces behind a transparent medium, but also the transparent medium or object, itself (Fig.1).

On this definition the air and the plate glass door are not perceptually transparent, unless for example there is fog in the air or marks on the door: thus physical transparency is not always accompanied by perceptual transparency. This statement is demonstrated also by the following simple experiment.

Take some black cardboard and fix on it with some glue a piece of transparent colored plastic (Fig.2). Provided

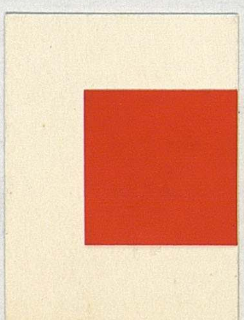

Fig. 2

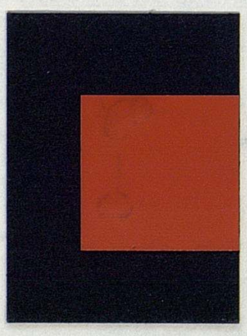

Fig 3

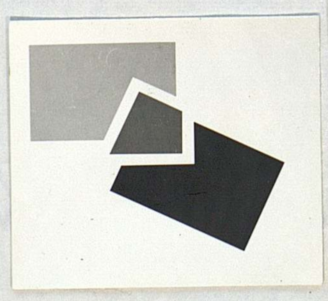

Fig. 4

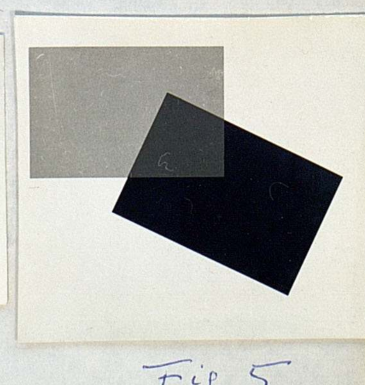

Fig. 5

that the layer of glue is homogeneous, you do not perceive the plastic as transparent: it appears to be an opaque piece of paper or other material. Changing the color of the cardboard (Fig.3) does not change the effect: the plastic still appears opaque.

Now let us do another experiment. Take three pieces of cardboard, one black, one dark gray and one light gray, and cut them according to Fig. 4; then put them together, as in Fig. 5. Physically it is a mosaic of opaque pieces of cardboard; but we perceive a rectangle through which we see a part of another rectangle. All the figures whose photographs are printed in this paper (except Fig.1,2, 3 and 6) have been constructed with this method, due to Professor W.Metzger; and although they are mosaics of pieces of opaque cardboard, many of them give rise to the perception of transparency. Thus, there are cases of physical transparency where perceptual transparency is absent (Fig.2, 3) and, on the other hand, cases where physical transparency is lacking and perceptual transparency is present (Fig.4, 5).

It should be clear then, that physical transparency is neither a sufficient nor a necessary condition for perceiving transparency; that is physical transparency cannot explain perceptual transparency.

The above conclusion may seem paradoxical, but it is, perhaps, more plausible when we remember that, as for every visual phenomenon, the causes of the perception of transparency have to be sought in the stimulation of the sense organ and in the physico-chemical processes in the nervous system to which the stimulation gives rise. It is a fact that the luminous flux reaches the cells sensitive to light in the retina only after having passed through several transparent media (the air and the transparent media of the eye, at least); but the visual apparatus does not receive the specific information for discriminating each physically transparent layer through which light has been filtered. The consequence of this state of affairs is clear: the perception of transparency cannot be explained by the physical phenomenon of filtration; it is a new fact originating in the optical sector of the nervous system as a result of special properties (to be investigated) of the distribution of the stimuli acting on the retinal cells.

One more proof that physical transparency is unessential for perceiving transparency: No attitude could support a perception of transparency in Fig. 2 or 3; but a simple change of the stimulating conditions causes the transition from opacity to transparency. Juxtaposing the figures in such a way that the borders of the sheets of plastic and of the pieces of cardboard coincide (Fig. 6), a transparent surface (corresponding to the plastic sheets) is perceived through which the white and the black of the pieces of card board can be seen.

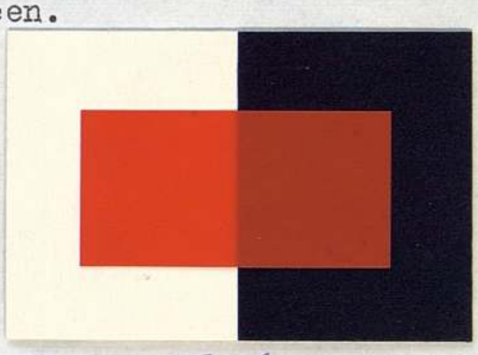

Fig. 6

This experiment shows also another fundamental characteristic of the perception of transparency. We may ask: which change in stimulation is responsible for the change from opacity (Fig. 2 and 3) to transparency (Fig. 6)? The local stimulation is the same in both situations: that is, the light reflected by the rectangular regions is the same. Therefore, the transparency phenomenon is not locally determined: it depends on a wider field, on the stimulation of several regions, as well as on their spatial and intensity relations.

2. The conditions of transparency

The conditions under which transparency is perceived ~~where~~ were discussed by Helmholtz (1867) and Hering (1888), and analyzed chiefly by Fuchs (1923) Tudor-Hart (1928) Heider-Moore (1933) Koffka (1935) Metzger (1953) Kanizsa (1955) and myself (1967, 1970). It is easy to demonstrate that there are two kinds of necessary conditions for perceptual transparency, <u>figural</u> and <u>color</u> conditions, since transparency can be abolished by altering either form (see Figures 7, 9, 11, compared with Fig. 8, 10, 12) or color (see Fig. 13 compared with Fig. 14).

Fig 7

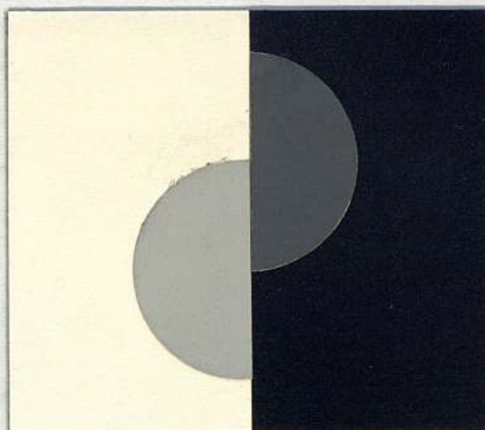

Fig. 8

Fig 9

Fig. 10

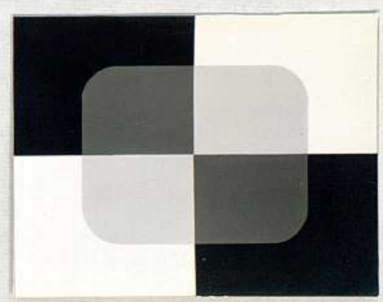

Fig 13

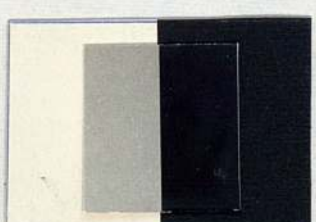

Fig. 11

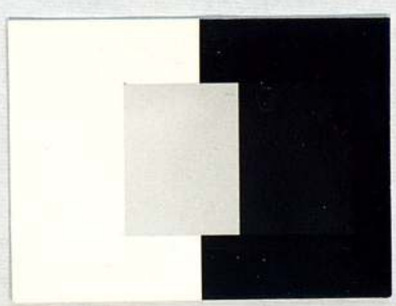

Fig. 12

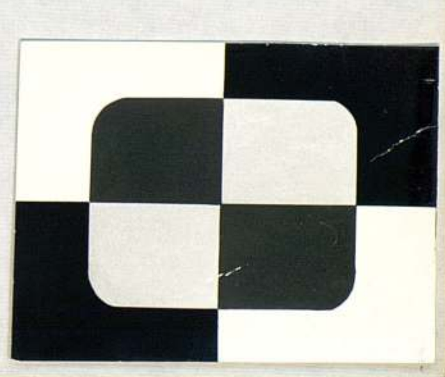

Fig 14

3. Transparency as ~~preceptual~~ scission

perceptual

Color conditions of transparency are particularly interesting because they have to do with the very nature of the phenomenon. In fact, the essence of the perceptual phenomenon of transparency consists in a <u>color scission</u> (G.Heider-Moore, 1933).

Let us try to describe what happens in Fig. 13 when we perceive transparency. The stimulation originated by the P region (see Fig. 15) has a remarkable effect, as it produces two different perceptual results: in fact we see an anterior layer T, which is transparent, and through it a second layer A*, having the same color as the contiguous layer A. (The same happens for the Q region, which also splits into the two regions T and B, but for the moment we'll confine our consideration to the left half of Fig. 13, that is to the A and P regions). The phenomenon can be described thus

$$\begin{tikzcd} & T \\ P \arrow[ur] \arrow[dr] & \\ & A \end{tikzcd}$$

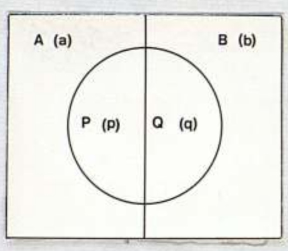

Fig. 15

FIG. 15

From now on a more precise meaning will be given to symbols. Capital letters will symbolize the various *areas* of the figures, while the corresponding small letters will symbolize *colors*. So, for example, A indicates the c - shaped area at the left of Fig. 13, while a indicates its color, namely white; A^* indicates the rectangular area of the same form and size of P and *perceived* behind it (the A^* layer is perceived as an uninterrupted continuation of the A region, without perceptual boundaries between them). The splitting process of the whole Fig. 7 is symbolized in Fig. 16.

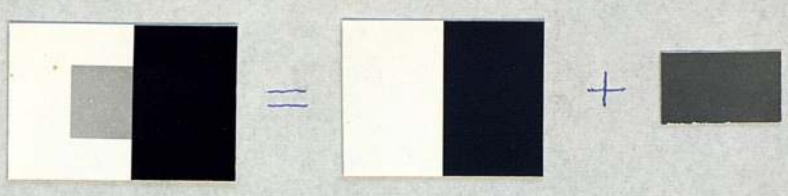

FIG. 16

As a further development in order to achieve a mathematical expression of perceptual splitting, we need small letters to symbolize not just colors, but more exactly their measurements.

It is well known that the description of a color ~~needs~~ requires three numbers, the so-called trichromatic coefficients, to be defined. But it is clear that a three-dimensional measuring system would bring with it a complexity which is not suitable to start a study. Therefore research in this direction had to be started with *achromatic* colors (the series of grays, from white to black) which vary only in the dimension of brightness and are defined by one number, the coefficient of *reflectance*.

The coefficient of reflectance gives a physical measure of the stimulus. This is necessary because a law of perceptual transparency, which is the main purpose of research in this field, must be the expression of a relation between one or more variables of stimulation and one or more variables of perception.

The luminous flux reaching a surface is partly absorbed and partly reflected. The ~~index~~ of reflectance coefficient

$$L = \frac{i}{I}$$

(where I is the light falling on a surface, i the light reflected by it) is simply the measurement of the percentage of light reflected by a surface. Therefore, an ideal white, reflecting 100% of the light falling on it, would have an ~~index~~ coefficient of reflectance of $\frac{100}{100} = 1$; an ideal black, which would absorb all of the light falling on it, without reflecting anything, would have a reflectance of $\frac{0}{100} = 0$. Thus the limits of the coefficient of reflectance are 0 and 1. In practice these limits are never reached: the white cardboard in our figures reflects 86% of the light (reflectance .86) and the black 4% (reflectance .04). The dark gray used in Fig. 7 has a reflectance of .14, and the light gray .36.

Now we are ready to resume the analysis of the perceptual color splitting phenomenon, which is the core of perceptual transparency. It has to be stressed that when transparency is perceived, not only the P region splits into two superimposed equal layers, but also the p color splits, giving rise to two colors, a and t, namely the colors of the two layers A and T. From now on we'll refer to p as the stimulus color, because it is the color to which the stimulation gives rise if no transparency is perceived, as in Fig. 17.

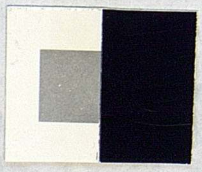

FIG. 17.

It has to be stressed that it is possible to say that the p color is replaced by colors a and t; but saying that p splits into a and p means asserting that there is a definite relation between p, a and t.

This problem among others has been treated in a ~~brillant~~ brilliant theoretical and ~~expreimental~~ experimental paper published by Grace Heider Moore under the supervision of Kurt Koffka several years ago. In her paper G.Heider formulated the hypothesis that the splitting colors are such that, fused together, they would again yield the stimulus color.

Koffka, in this treatise, puts the question in very simple terms. It is well known that by fusing together appropriate amounts of blue and yellow, with the method described below, the resulting mixture-color is gray. Therefore, when transparency is perceived, if the stimulus color is gray and the conditions require that one of the scission colors be blue, then the other scission color must be yellow. Symbolically, if $Y + B = G$, then $G - B = Y$.

The same statement, applied to the left half of our model (Fig. 13 and 15) would be: when transparency is perceived, the stimulus color p being light gray, and one of the scission colors, a, being white, the other scission color (t, the color of the transparent layer) has to be a darker gray, as a fused with t has to result again in p.

But what will be the quantitative expression of t? It is clear that the theory is not a quantitative one. Let us see if, confining ourselves to achromatic colors, it is possible to make it quantitative.

4. Talbot's law and the equations of transparency

The starting point for the development of a quantitative law of color scission, which will allow us to predict exactly the scission colors, given the stimulating conditions, has been the law of color fusion, known as Talbot's law.

A very simple apparatus for fusing colors is the "color wheel". To use this device we take two pieces of circular, colored paper, for example one light gray and one dark gray, we cut them such that the resulting sectors sum to 360°, and we attach these to the color wheel. In Fig. 18, for example, the two sectors are equal, each being 180°, while in Fig. 19 the lighter one covers $\frac{3}{4}$ (270°) of the circle, and the darker $\frac{1}{4}$ (90°). We'll call the lighter sector "A", and its reflectance "a", and the darker sector "B", with its reflectance "b".

When the color wheel turns at a high speed, a given region of the retina is stimulated alternately by the light reflected from the A and from the B sectors. The rotating disk is then perceived as homogeneous and at rest, and its brightness lies between the brightness of the two components.

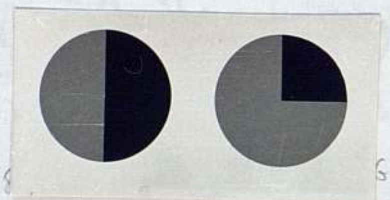

FIG. 18   G. 19

Fig 18   Fig 19

Talbot's law asserts that in the special case of two equal sectors (Fig. 18), the reflectance c of the fusion color is the simple average of the reflectances of the components, that is

$$c = \frac{a + b}{2}$$

So, f.ex., if a = .60 (light gray) and b = .20 (dark gray)

$c = \frac{.60 + .20}{2} = .40$ a gray whose reflectance is between a and b.

In the case of Fig. 19, a further datum has to be taken into account. Here the two colors are weighted differently, because the retinal region where the image of the rotating disk is projected is stimulated $\frac{3}{4}$ of the time by the light reflected from the A sector and $\frac{1}{4}$ of the time by the light reflected from the B sector, therefore the measure of the reflectance ~~C~~ of the fusion color is obtained by giving the c component color a a weight which is triple the weight given to color b, that is

$$c = \frac{3}{4}a + \frac{1}{4}b \quad \text{or} \quad .75\,a + .25\,b$$

and given the reflectances .60 and .20

$$c = (\tfrac{3}{4})(.60) + (\tfrac{1}{4})(.20) = (.75)(.60) + (.25)(.20) =$$

$$= .45 + .05 = .50$$

More generally, if α is the proportion in which color a has been taken and β the proportion of color b, Talbot's law is expressed by

$$c = \alpha a + \beta b$$

and, since $\alpha + \beta = 1$ (the two sectors into which the disk is divided, as in our example $\frac{3}{4}$ and $\frac{1}{4}$ or .75 and .25, sum up to 1), so that $\beta = 1 - \alpha$, the above equation can also be written

$$c = \alpha a + (1 - \alpha)b$$

where α and $(1 - \alpha)$ are the proportions in which the two components are present in the fusion.

Now with a quantitative law for color fusion in hand, which allows us to predict the reflectance of the fusion color from the reflectances of the component colors in their weighted proportions, the next step is very simple. According to Heider and Koffka's ~~Theory~~, the same relations exist between scission colors and stimulus color as between component colors and fusion color; so the same equation can be used to describe both processes. Therefore changing symbols according to Fig. 15, the equation describing the scission process in ~~trasparency~~ will be theory transparency

$$p = \alpha a + (1 - \alpha)\, t \qquad (1)$$

Before using the above equation for further developments, it seems opportune to make clear the meaning of the symbols involved: p is the reflectance of one of the two regions where the splitting and therefore the transparency process takes place (the P splitting region contiguous to the A region); a and t are also reflectances, namely the reflectances of the two splitting surfaces, A^* and T. But they differ in one important respect. a like p is one of the data of the problem. We can measure the reflectance of the P region with a photometer or choose at will the reflectance of P, f. ex. selecting for P a paper of known reflectance. The same can be done with a, because the color of A^*, the splitting surface, is the same as that of the contiguous region A, whose reflectance can also be chosen at will. But for t there is no such possibility, as it is not one of the starting data, but an effect of the "constellation" of stimuli acting on the eye, and it varies in a determinate way with them, (provided that the constellation of stimuli generates transparency).

The meaning of the symbol α is more difficult to clarify. In the process of color fusion, α and $(1 - \alpha)$ are the proportions of the two components which, stimulating the eye, give rise to the fusion color. Applying the the hypothesis of the parallelism between fusion and scission processes, in the process of color scission α and $(1 - \alpha)$ are the proportions of the stimulus color which are attributed to each of the scission colors. As in the case of fusion where two component colors could contribute to the process with different weightings (f. ex. $\frac{1}{4}$ and $\frac{3}{4}$), so in the case of transparency the scission colors may assume different proportions of color (for ex., also in this case, $\frac{1}{4}$ and $\frac{3}{4}$).

What will the effect of this unequal distribution be? Let us consider, first of all, the transparent layer. What happens when we put more and more of a soluble, colored material into a glass filled with water? It is evident that the more color - be it dark or light - we put in, the less transparent the liquid. And, of course, the objects seen through the liquid become less and less visible. So, the effect of a color distribution, in which the transparent layer receives more and more color and the opaque layer less and less, seems to be a progressively decreasing transparency of the transparent layer and a correspondingly decreasing visibility of the opaque layer seen through the transparent one. And as α and $(1 - \alpha)$ are the coefficients which describe the proportion of color going respectively to the first (opaque) layer, and to the second, transparent layer, we can complete our description noting that when α is small and $(1 - \alpha)$ large, then visibility (or intensity) of the first layer is low, and transparency of the second layer is (correspondingly) low; while, if α is large and $(1 - \alpha)$ is small the contrary is true.

This statement can be checked with equation (1). Putting $\alpha = 0$ the equation ~~symplifies~~ to $p = (0)(a) + (1-0)t$, that is, simplifies $p = t$, which means that all the color of P goes to T, A^* becomes invisible and there is no transparency (Fig. 20). On the contrary if $\alpha = 1$ the equation ~~symplifies~~ $p = (1)(a) + (1 - 1)t$, simplifies that is $p = a$, all the color of P goes to A^*, T becomes invi sible, that is, perfectly transparent as the air (Fig. 21).

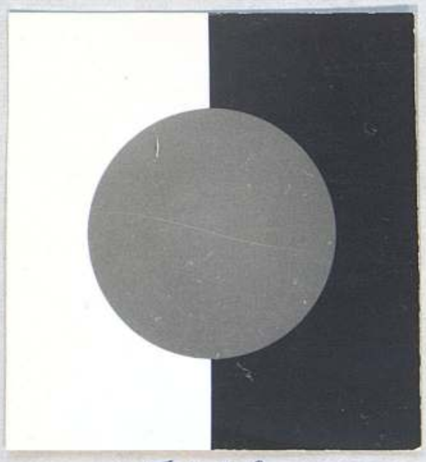
Fig. 20

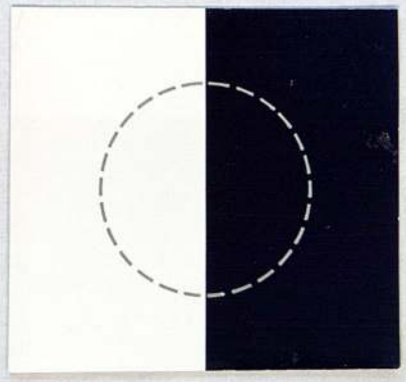
Fig. 21

These are the two limiting cases. Perceptual transparency, where an object is seen through another object, takes place only when the value of α is between 0 and 1.

Now we are able to understand the meaning of α. Having the lowest possible value, zero, when there is no transparency, the highest possible value 1 when transparency is perfect, and the intermediate values, growing with growing transparency, it is, obviously an index of transparency.

At this point the meaning of the symbols used in the equation (1) should be clear: *p* and *a* are the data of the problem, the ~~independ~~ variables, which can be varied at will, independent while α and *t* are the dependent variables, on which the effect of the variables can be tested. It is interesting to be able to predict these effects, and equation (1) should offer us this opportunity.

But from elementary algebra we know that an equation in two unknowns (α and *t*) is indeterminate, having an infinite number of solutions. However we have used till now only one half of the model (Fig. ~~9a~~). The other half allows us to wri- 15 te an analogous equation (still on the basis of Talbot's Law) because just as the *p* color (in the case of transparency) is distributed between *a* and *t*, the same happens for the *q* color in relation to *b* and *t*.

Therefore we can write

$$q = \alpha' b + (1 - \alpha')t' \qquad (2)$$

The reason for using α' and $\underline{t'}$ instead of α and $\underline{t}$, is that transparency, and also color of the transparent layer, may be unequal in the P and Q regions.

In fact, in general no difference in transparency, and/or color of the T layer, in the P and Q regions is described by subjects (balanced transparency). But there are also cases where a clear difference, at least in degree of transparency is perceived (unbalanced transparency) and even cases where transparency is perceived only in one of the P and Q regions (partial transparency). However, referring to the cases of balanced transparency, where $\alpha = \alpha'$ and $t=t'$, equations (1) and (2) are a system of two equations in two unknowns, which is determinate and whose solutions are

$$\alpha = \frac{p - q}{a - b} \quad (3) \qquad\qquad t = \frac{aq - bp}{(a+q)-(b+p)} \quad (4)$$

5. <u>Some necessary conditions for perceptual transparency</u>

At this point it is quite natural to ask what is the contribution of the above mathematical developments to our understanding of the perceptual phenomenon we are studying.

The advantages of having solved the system of two equations are the following: equations (3) and (4) state the relations between stimulating conditions (the reflectance of the 4 regions determining the phenomenon) and the two perceptual characteristics of the ~~phenomeno~~, the degree of transparen- phenomenon

. cy[1] and the color of the transparent layer; in other words, they describe precisely the relations between the ~~indipendent~~ independent and dependent variables. They offer the possibility of predicting the above perceptual phenomena from the measures of the stimulation .

Besides, the equation for α , given its great simplicity, can be used for drawing inferences which give the opportunity of empirical test.

It has been shown above, that the acceptable values of α are between 0 (no transparency) and 1 (perfect "transparency" or invisibility of the upper layer). This allows us to draw the following inferences:

A. If $\alpha = \dfrac{p - q}{a - b}$ has to be less than 1, then the numerator of this fraction has to be less than the denominator: that is <u>the difference in reflectance between the p and the q regions, has to be less than, the difference in reflectance between the a and the b regions, otherwise transparency is not possible</u>.

We can easily check this prediction perceptually. In Fig. 7, 9 and 13, <u>p</u> and <u>q</u> are two different shades of gray, while <u>a</u> and <u>b</u> are white and black. Then in all these cases, where transparency is perceived, the above necessary condition for transparency is respected, because the difference in reflectance between a light and a dark gray is clearly less than the difference in reflectance between white and black.

On the other hand, let us construct a figure where A and B are respectively light gray and dark gray, and P and Q,

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(1) It should mentioned that besides the perceptual scission measured by α there is yet another factor which contributes to the perception of transparency, namely the color of the transparent layer. A darker transparent layer, for a given proportion of color, is perceived as more transparent than a lighter colored layer.

respectively white and black (Fig. 22). In this case (p - q) is greater than (a - b); and in fact the central rectangle corresponding to the P and Q regions is opaque. Therefore the algebraic deduction finds its confirmation in the perceptual experience.

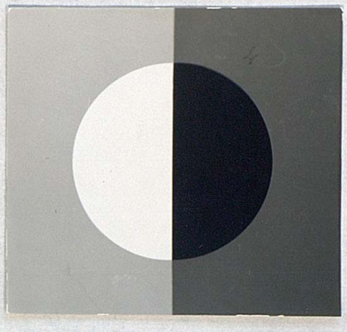


Fig 22

With a slightly more complex pattern we can see yet something more. Fig. 23 is generally perceived as a checker board with a transparent strip superposed ~~in~~ its central part. As on can be seen, the central part of the figure (whose first two rows are reproduced in Fig. 24) is an alternation of the sequence A P Q B with the inverse sequence B Q P A, thus being a replication of the structure of Fig. 13; while the white and black squares, continuing the sequences at both sides in Fig. 23 have only a stabilizing function.

In Fig. 23 and 24 the above deduced necessary condition for transparency is respected: the difference in reflectance is less between the splitting regions P and Q than between the A and B regions. If, using the same model, this relation is inverted, as in Fig. 25, an interesting aspect, which was already present, though not so clearly, in Fig. 22, is perceived. Not only, as in Fig. 22, has the central part become opaque, but the sides of the checker board have become transparent. In fact, the reflectance relations, which in this case correspond to the brightness relations, decide which regions split perceptually and which not; in other words, the stimulus relations and not the experimenter decide which regions acquire the func color cover

Fig 23

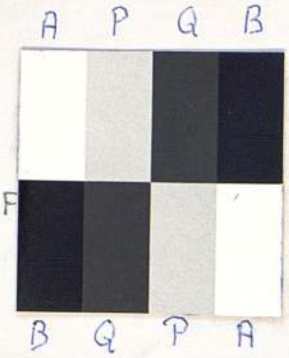


Fig. 24

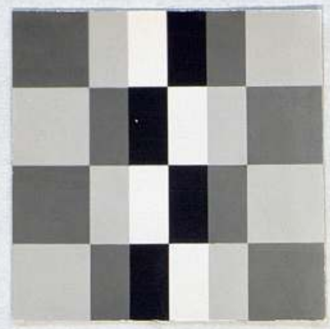


Fig. 25

tions of P and Q, and which of A and B.

B. If α is an index of transparency, transparency should increase as α increases.

This prediction can be easily tested. Let us construct two models, where A and B are respectively white and black, so that (a - b) is equal in both. In one of the models (Fig. 26) P and Q are very similar in brightness, while in the other (Fig. 27) P and Q are very dissimilar (but less dissimilar than A and B).

As fractions with equal denominator (a - b) increase with the numerator, the prediction is that in Fig. 26, where the value of (p - q) is small, there should be relatively little transparency, while in Fig. 27, where the value of (p - q) is great, transparency should be greater.

The reader can verify for himself, that both predictions are fulfilled.

Fig. 26

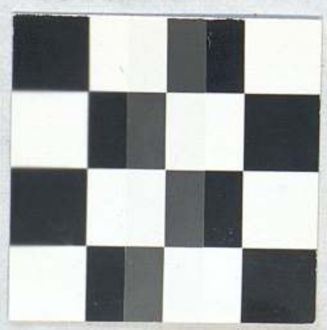


Fig. 27

C. A further prediction, equally testable, can be drawn from the requirement that α be greater than zero. In fact, if α beca me negative, the opaque layer seen through the transparent layer would have to receive a negative quantity of color, which is devoid of meaning.

As $\frac{p - q}{a - b}$ has to be greater than zero, i.e. positive, in order for transparency to be perceived, then the numerator and the denominator of the fraction have to be either both positive or both negative. And as (p - q) is positive if p is greater than q, and negative if p is smaller than q and the same is true for (a - b), the above necessary condition follows: either $p > q$ <u>and</u> $a > b$, or $p < q$ <u>and</u> $a < b$. In both cases α is positive.

Clearly $(p > q)$ means that the reflectance of the P region is greater than the reflectance of the Q region; but the same relation appears, perceptually, as a brightness relation. In other words we may interpret $(p > q)$ as "the P region is brighter than the Q region". Therefore the above necessary condition for perceiving transparency can be expressed in a perceptually meaningful form as follows: if the P region is brighter than the Q region, then the A region has to be brighter than the B region. If the P region is darker than the Q region, then the A region has to be darker than the B region. Otherwise transparency is not possible.

As we have regularly called A the brighter of the A and B regions, the above condition can be checked by noting that in our figures, where transparency is present, P is brighter than Q.

As a further check Fig. 28 has been constructed, where P is darker than Q, while A is brighter than B. Here, transparency has never been perceived.

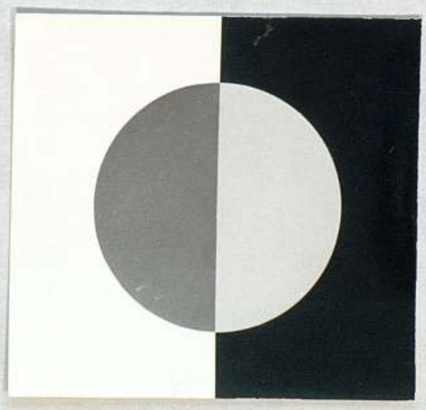


Fig. 28

6. Predictions from brightness relations

While the α equation offers several ways of testing inferences perceptually, the same seems not to be true for the t equation, whose algebraic relations don't seem to offer a perceptual interpretation. However, following another path it has been possible to infer perceptually meaninful algebraic relations between stimulus conditions and the color t of the transparent layer.

To avoid the long process of formal deduction here, we will follow an intuitive short cut.

It has been shown that translating Heider and Koffka's theory into quantitative terms (equation 1), the reflectance of the stimulus color p is equal to the weighted average of the reflectances of the scission colors a and t.

But if p is the average of a and t, it is clear that if a is greater than p, then t has to be less than p; and if a is less than p, then t has to be greater than p. We can represent these relations graphically in the following way, where the height corresponds to the amount of reflectance.

Either $\begin{tikzcd}[row sep=small, column sep=small] & t \\ p \arrow[ur, dash] \arrow[dr, dash] & \\ & a \end{tikzcd}$ or $\begin{tikzcd}[row sep=small, column sep=small] & a \\ p \arrow[ur, dash] \arrow[dr, dash] & \\ & t \end{tikzcd}$ that is, either $\cancel{t} > p > a$ t

or $a > p > t$.

It has to be added that the above statement is valid also for a weighted average, if the weights are positive and sum to 1, as α and $(1 - \alpha)$.

But so far we have considered only the left half of our model of Fig. 9a. It is clear that for the right half, following the same reasoning, we can state that either $\begin{tikzcd}[row sep=tiny, column sep=small] & t \\ q \arrow[ur, dash] \arrow[dr, dash] & \\ & b \end{tikzcd}$ or 15

$\begin{tikzcd}[row sep=tiny, column sep=small] & b \\ q \arrow[ur, dash] \arrow[dr, dash] & \\ & t \end{tikzcd}$, that is, either $t > q > b$ or $b > q > t$.

Now, having two acceptable ordered series of reflectances for the left part of the model and two for the right part, we can obtain the following four combinations; that is, four complete models

	left half		right half
1	$t > p > a$	and	$t > q > b$
2	$t > p > a$	and	$b > q > t$
3	$a > p > t$	and	$t > q > b$
4	$a > p > t$	and	$b > q > t$

[~~Trying to do this~~] it becomes clear that there are several reflectance sequences corresponding to combination 1.| Among them we select the following two, which also correspond to the above-stated necessary conditions of transparency, namely *

|From

A. $t > p > a > q > b$ (Fig. 29)

B. $t > p > q > a > b$ (Fig. 30)

In addition, there is the single reflectance sequence(1) corresponding to both the combinations 2 and 3, namely

C. $a > p > t > q > b$ (Fig. 31)

And we select again among the several possible reflectance sequences, the following two, corresponding to combination 4, namely

D. $a > p > b > q > t$ (Fig. 32)

E. $a > b > p > q > t$ (Fig. 33)

---

(1) Sequence C has been drawn from combination 3; the sequence $b > q > t > p > a$, corresponding to combination 2, corresponds exactly to C if instead of having ~~a~~ $> b$ and $p > q$ we define $b > a$ and $q > p$. a

substitute * [In trying to construct patterns corresponding to the models]

It has to be stressed that an ordered series of reflectances, as the A,B,C,D,E sequences, corresponds perceptually to an ordered series of brightnesses. Thus the interest of the above deductions becomes clear. Each one of the above sequences states a relation between stimulus conditions (brightness order of the APQB regions) and brightness of the transparent layer T which can be tested perceptually. So, for example, sequence A states that if the order of brightness of the stimulus colors is p > a > q > b, then the transparent layer t is the ~~brightness~~ brightest of all. In other words, as can be seen in Fig. 29 from the order of brightness of the four regions the brightness of the transparent layer t, that is, its position in the above order, can be predicted [1].

Fig. 29

Fig 30

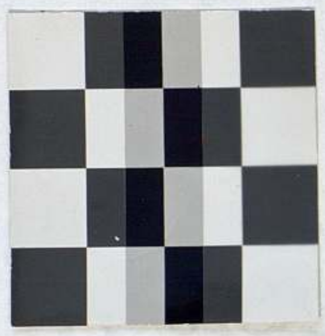


Fig 32

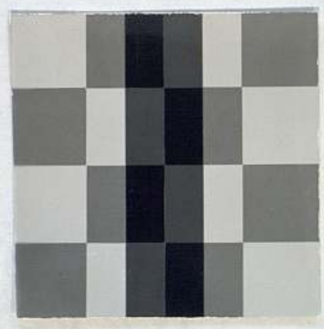


Fig 33

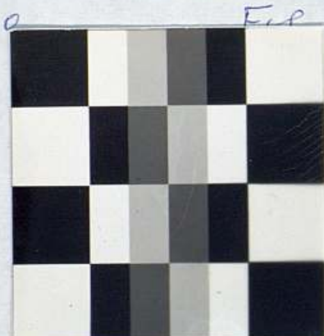


Fig. 31

(1) As for the differences between the transparent layers in Fig. 29 and 30 on the one hand and 32 and 33 on the other, it has to be kept in mind that these figures differ in transparency: a black veil which is very transparent differs very much in its aspect from a black and very thick veil.

7. Analytic representation

An analytic representation developed by C.Remondino, gives us the opportunity of further ~~clarify~~ the above relations and of discovering new aspects of this field of research. clarifying

As four variables, a, p, q, b, cannot be represented on a plane, the system had to be simplified, giving fixed arbitrary values to the apparently less important variables a and b. Besides it has been stated by definition that $a > b$.

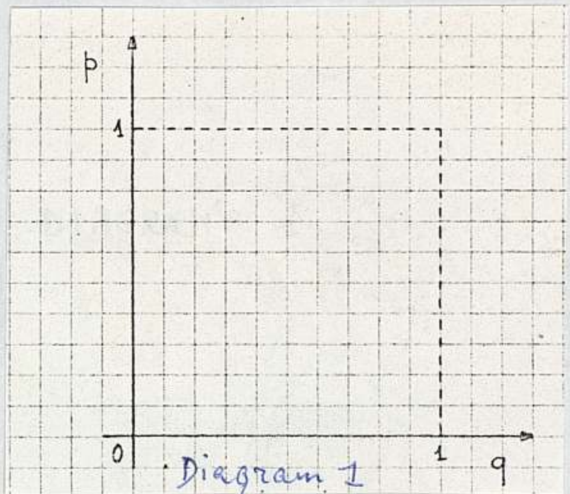


Diagram 1

Thus p and q can be taken as cartesian coordinates varying between 0 and 1. Every point of the square in diagram 1 corresponds to a different pair of p and q values.

The λ and t equations can therefore be considered as functions of the p and q variables, a and b being two constants of known value.

Now let us give the λ equation the limiting values 0 and 1, in order to see which p and q values satisfy these conditions. For $\lambda = 0$ we get $p=q$, while for $\lambda=1$ we obtain $p= q + (a - b)$. In Diagram 2 we see that $p=q$ is represented by the diagonal from the lower left corner to the upper right.

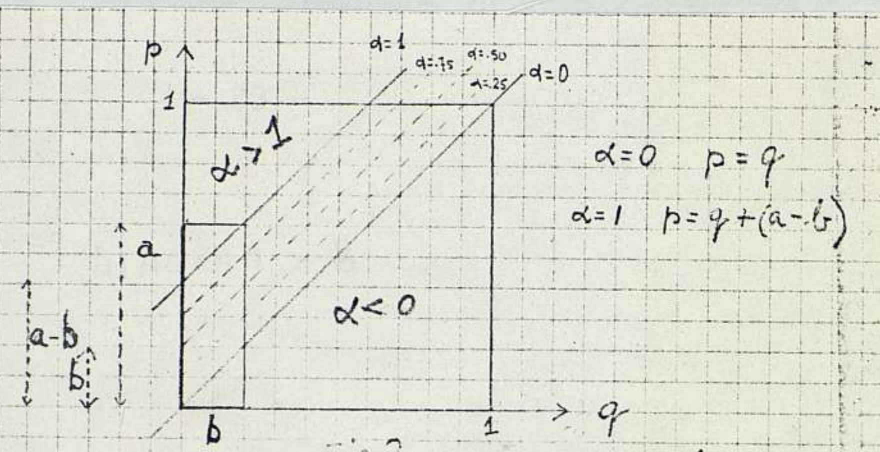


Fig 2
Sizes of p and q for admissible values of α
(α>0 and α<1)

Diagram 2

It is clear that every point on this line represents an equal value of p and q: from the lower left corner, where p=0 and q=0 to the upper right where p=1 and q=1.

To represent the p and q values for $\alpha=1$ it is necessary to choose the values of the a and b constants. The values of p and q when $\alpha=1$ follow a line which is parallel to the p=q line: in fact, in this case, p differs from q by a constant value.

In diagram 2, a and b are also represented, in order to note the special point on the $\alpha=1$ line, where p=a and q=b. The importance of this point will become clear in the next diagram. It is, however, evident that in this special case the α equation reduces to

$$\frac{a - b}{a - b} = \frac{p - q}{p - q} = 1$$

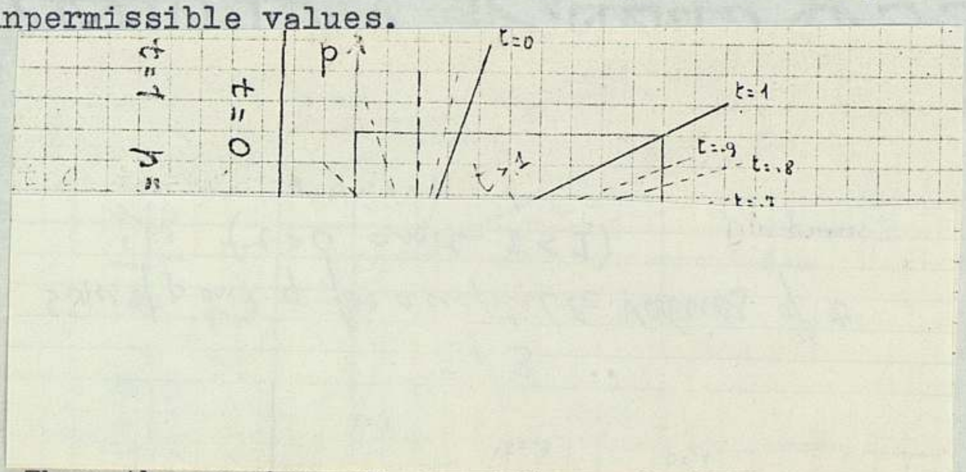
But this is not the only such case, since it is sufficient that the two differences p - q and a - b are the same, in order to have $\alpha = 1$.

With this last operation a region of the diagram, the strip limited by the two lines obtained by setting $\alpha=0$ and $\alpha=1$, corresponds to all the admissible values of α, that is the pairs of p and q values corresponding to the points on this strip constitute all the values of between 0 and 1, when the a and b values correspond to those of the diagram.

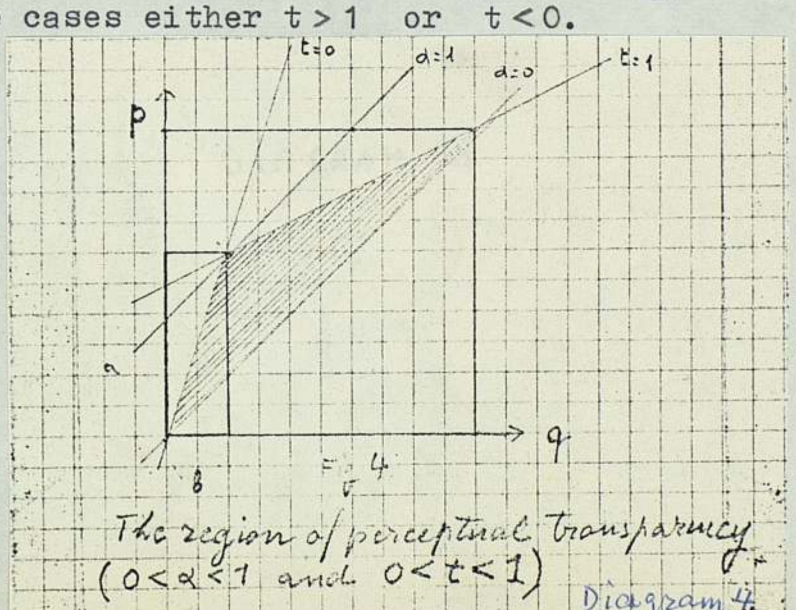
In the next diagram the same operations, determining the region of permissible t values, have been performed.

However, the results are not as simple as with the α equa

tion: when $t=0$, $p=\frac{a}{b}q$ and when $t=1$, $p=\frac{a-1}{b-1}q-\frac{a-b}{b-1}$.
Lines joining the p and q values which give as a result $t=0$ and $t=1$ have been drawn, as well as some other lines corresponding to intermediate values of <u>t</u> (as was done for α in the preceding diagram). The resulting region of <u>p</u> and <u>q</u> values giving permissible <u>t</u> values spreads over the diagram leaving only two unoccupied sectors, as regions of unpermissible values.



Thus the region of p and q values giving admissible α <u>and</u> t values has been circumscribed (Diagram 4). By superimposing the two preceding diagrams (2 and 3) a triangular region appears. Its longer side corresponds to the above described diagonal, and its opposed vertex, to the $p=a$, $q=b$ point. It follows that this is the only admissible case where $\alpha=1$, because all the remaining parts of the $\alpha=1$ line fall in the inadmissible region of <u>t</u>, that is, in all these cases either $t>1$ or $t<0$.



The region of perceptual transparency
$(0<\alpha<1$ and $0<t<1)$

Diagram 4.

However, the results are not as simple as with the α equation: when $t=0$, $p=\frac{a}{b}q$ and when $t=1$, $p=\frac{a-1}{b-1}q-\frac{a-b}{b-1}$. Lines joining the p and q values which give as a result $t=0$ and $t=1$ have been drawn, as well as some other lines corresponding to intermediate values of $\underline{t}$ (as was done for α in the preceding diagram). The resulting region of $\underline{p}$ and $\underline{q}$ values giving permissible $\underline{t}$ values spreads over the diagram leaving only two unoccupied sectors, as regions of unpermissible values.

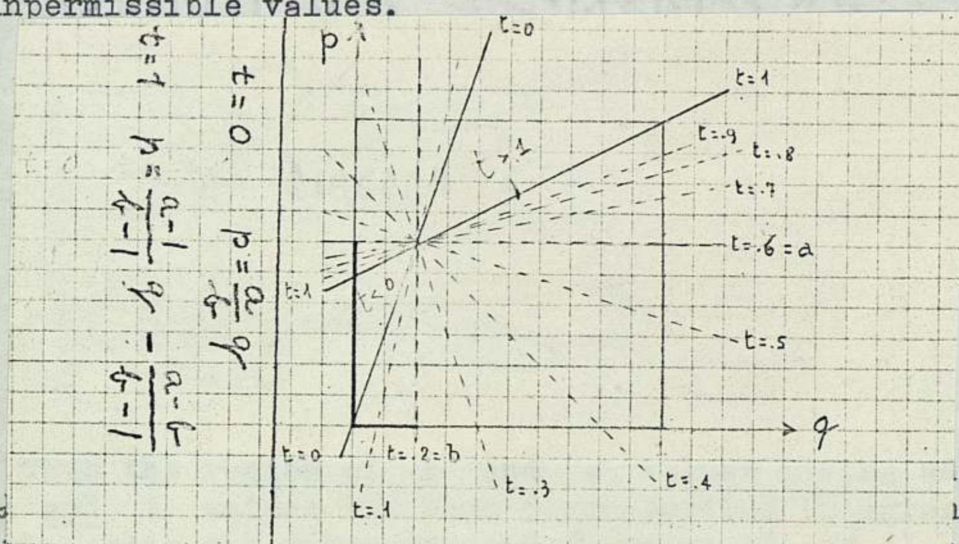


Fig. 3

Sizes of p and q for acceptable values of t

($t>0$ and $t<1$)

Diagram 3

... missi-
ble α ... 4). By
supe... trian
gula... the
abov... the
p=a, ... missible
case... the $\alpha=1$
line fall in the inadmissible region of $\underline{t}$, that is, in all these cases either $t>1$ or $t<0$.

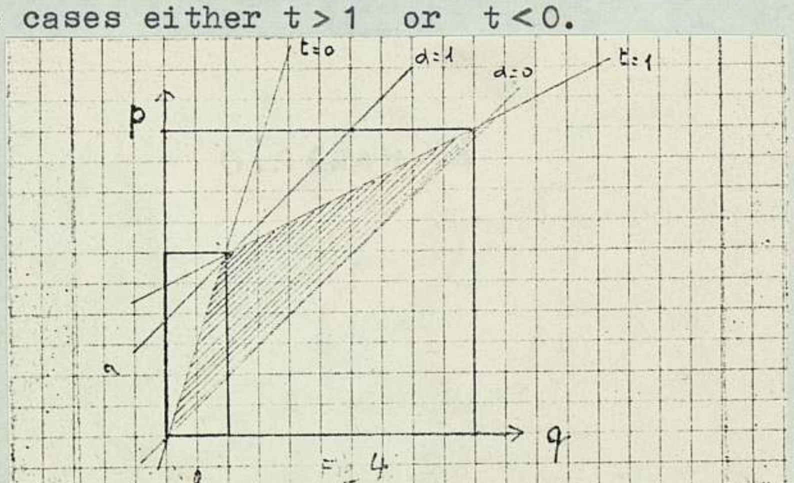


Fig. 4

The region of perceptual transparency

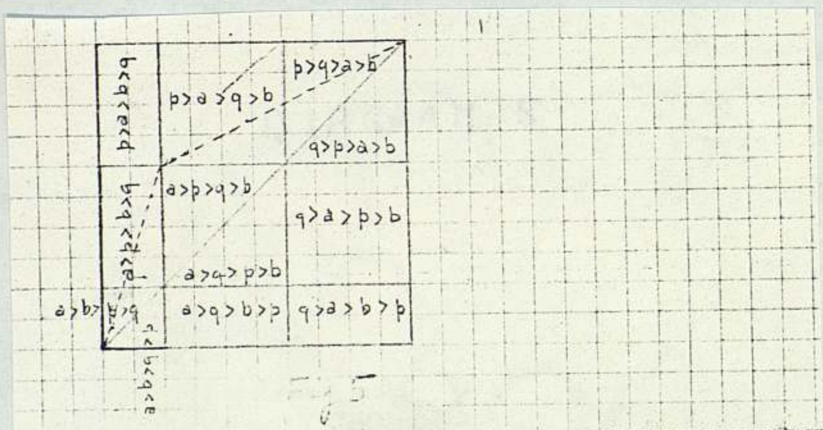
($0<\alpha<1$ and $0<t<1$)

Diagram 4.

But what is the value (or the color) of t when α =1? From the diagram we see that it could be any color, because at this point all the lines corresponding to different t values, from 0 to 1, (and also the inadmissible ones) are converging. The color seems to be indeterminate, and in fact, the solution of the t equation for a=p, b=q is $\frac{0}{0}$. The solution is a meaningfull one, because when transparency is perfect (α =1) the t layer disappears and its color is indeterminate.

The size and, to some extent, also the shape of the triangular region [which, as has been said, can be cho~~o~~sen at will vary with the a and b constants.] As the difference between a and b increases, the relative size of the region where α and t are greater than zero and smaller than 1, namely the region of acceptable values of α and t, also increases. If a=1 and b=0, (that is if a is a perfect white and b a perfect black), then the above region coincides with the half square above the diagonal. *

Diagram 5 shows the reflectance relations (corresponding to the brightness relations) between a, b, p, q, in the different regions of the diagram. It is interesting to notice that only the a> p > q >b region is completely included in the region of acceptable α and t values. The other regions, whose brightness relations have been used for drawing inferences about t, are each only partly included in this region.



Brightness relations between A P Q B regions compatible with the perception of transparency
Diagram 5

* [vary with the a and b constants (which, as has been noted, can be chosen at will)]

It is clear that, according to the theory, the region of acceptable <u>t</u> and α values should be the region where transparency is possible, since the necessary chromatic conditions for transparency are present.

But what if transparency happens to appear in one of the "forbidden" regions? What if, for ex. transparency should be perceived when <u>a</u>, <u>p</u>, <u>q</u>, <u>b</u> measures were such that $\alpha > 1$? It might seem that this would be proof that the whole theory is invalid, since this would require that one of the layers receive a negative quantity of color. But it has to be remembered that the system of two equations, (1) and (2), can be solved only on the condition that $\alpha = \alpha'$ and $t=t'$, that is, when the degree of transparency and "color" of the transparent layer are equal in the P and Q regions. As a matter of fact, t and/or α outside the acceptable limits means only that there are no values of α and t inside these limits which satisfy the above system of equations, or, in other words, that there is no value for α and/or for t which is equal for the P and Q regions.

Thus, if in these cases transparency is perceived, it is not the form of <u>balanced</u> transparency for which the α and <u>t</u> equations are valid. As a matter of fact cases where transparency is unbalanced ($\alpha \neq \alpha'$ and/or $t \neq t'$) also exist and they appear in the "forbidden" regions of the diagram. That in these cases transparency is unbalanced is clearly perceived. The most typical case is the extreme one, of <u>partial</u> transparency, where transparency is perceived only in either the P or the Q region, the other region being perceived as opaque (Fig. 34). It is clear that for these cases the α and t equations are not valid, and new equations have to be developed.

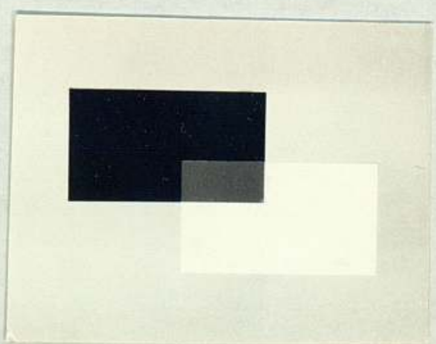


Fig. 34

truism

suff.

Transparency

1. Transparency: a perceptual problem?
   physical transparency neither sufficient nor necessary condition
   conditions of perc. transp. not local: interaction between differently stimulated fields

suff. see

2. The sensory problem: Helmholtz, Hering
3. Why working with apparent instead of real transp. (more freedom in varying stimulation) more easy to see (not to hit) variables
4. Conditions of perception of transparency (figural, color) Here colour
5. Type of research: not conformistic (at least not the traditional way for Psychology)
   good continuation
   not hypothesis collection of facts (or measurements) → diagram → → fit curve → equation → theory
   but Theory or hypothesis → algebraic expression → algebr. manipulating more suitable one → deducing necessary conditions of phenomenon → control on the facts
6. Colour conditions. Definition of colour as a stimulus: 3 numbers
   Achromatic colors. 1 number: reflectance or albedo

7. Φ standard model people in general perceive transparency
   A B P Q a p q b

   transparency as perceptual scission

   $P<^{A}_{T}$ one sort of stimulation — 2 effects

   Stimulus colour = region seen in isolation

8 Relation between stimulus colour and 2 scission colours
The Heider and Koffka solution

8. The Heider and Koffka solution 2

Scission colours are such that, if mixed, they give again the stimulus color

If $Y + B = G$ than $G - B = Y$

Not algebra. Possibility of having an algebraic quantitative law of colour scission as there is a quantitative law of color mixture.

9 Talbot's law (color wheel mixture) - Newton
{Vedere come l'ha espressa Newton}

$$\alpha a + (1-\alpha) b = c$$
$$\tfrac{3}{4} \cdot .8 \quad \tfrac{1}{4} \cdot .4 \quad = .6 + .1 = .7$$

α } proportions summing up to 1
$(1-\alpha)$ }

10. ~~Same~~ law describes color scission. Using symbols according to our models

$$p = \alpha a + (1-\alpha) t$$

proportion (quantity) of color going to tr. and the other layer
intensity / visibility — intensity or transparency

each interpretation with equation

$\alpha = 0$ $p = t$ no transparency

$\alpha = 1$ $p = a$ perfect transparency

limiting cases of no scission. Real transparency $0 < \alpha < 1$

α coefficient of transparency

but transparency depends also on color of transparent layer (D'Zmura's work)

11. Meaning of every symbol clear

a, p known quantities = independent variables

α, t unknowns : eq. indeterminate

Seemingly blind alley
so far only $\frac{1}{2}$ of the model was used
But other equations. If $t = t'$ and $\alpha = \alpha'$
as in our examples it seems to be the case

$$\alpha = \frac{p - q}{a - b} \qquad t = \frac{aq - bp}{(a+q) - (b+p)}$$

2

What use? Apart from satisfaction of having been successful in solving the eq.

Deducing necessary conditions

12. <u>Check the deduced formulas</u> 3

See if there is correspondence

<u>α between 0 and 1</u>. Limiting cases - no fission

$\alpha > 1$, $\alpha < 0$ otherwise one or the other layer would receive a negative quantity of color → devoid of meaning.

<u>1.</u> $\alpha < 1$ numerator less than denominator

~~$|p-q| <$~~ $\boxed{|a-b| > |p-q|}$ (otherwise $\alpha > 1$)

difference in reflectance has to be greater

Examples a, b colours more different than $p-q$

<u>Little difference</u>: p and q similar

α near to 0 little transparency

<u>great difference</u>: p and q very unsimilar

α near 1 great transparency

<u>2.</u> $\alpha > 0$

numerator and denominator both positive or both negative

$\boxed{\text{If } p > q \text{ then } a > b}$ defining $a > b$

(If $p < q$ then $a < b$)

Example.

—

Same reasoning possible with the <u>second equation</u> (other necessary conditions); but no perceptual meaning has been found till now for ~~[illegible]~~ complicated expression

13 However it has been possible through another way to make some interesting predictions about the color t of the transparent layer.

Lack of time - give up the rigorous deduction

Let us start intuitively

$p \begin{smallmatrix} \nearrow a \\ \searrow t \end{smallmatrix}$ $q \begin{smallmatrix} \nearrow b \\ \searrow t \end{smallmatrix}$

I half — II half of the model

Ia $a > p > t$ — IIa $b > q > t$

Ib $t > p > a$ — IIb $t > q > b$

Taking into account the necessary conditions

Associating each I half condition for with each cond. for II half ~~order of brightness~~ condition the following ~~orders of brightness~~ combinations brightness sequences are obtained

if $q > p$ and $q > t$, then t midway between p and q and ~~the model is transparent~~ transparency occurs,

The ~~meaning~~ result of this operation is that than ~~knowing~~ from the ~~order~~ rank order of reflectance of the a p q b areas ~~it is possible to predict~~ the place, in this rank order, of the reflectance (that is of the brightness) of the transparent layer can be predicted

In other words

14. D. Stefano

Study of the "~~natural~~ specific transparency" of achromatic colours

Which are the openings of the episcotister giving same transparency for different shades of grey

linear func. for log t

different for different grounds

~~now re following study~~ :

15 Remondino. member of the staff. Not regular distinguished psychologist and good mathemat. After having read a first draft of the my transparency study he sent me a very clarifying analytic study of the tr. equations.

I Parte ricerca

II Ricerca d'Urso

III Remondini

IV chaos

V L'uovo di Colombo

VI La trasparenza parziale

VII Prospettive di sviluppo

infer deduction

prejudice fase = stage

retta straight line

explicitation intuitively

necessary - sufficient condition

Vedere se funziona la cond. necessaria p. la trasparenza parziale nella pp. [II]

6

1. Transparency a perceptual, not a physical problem

(relation betw physical and perc. tr.)

2. Hystori~~cal~~ of the problem Hering Helmholtz transparency as physical problem

3. Why working with illusory and not with real tr.

We are looking ~~about~~ for conditions of stimulation of the eye giving transparency as a result: it is more simpler to ~~st~~ use papers of known reflectancy than filters.

1 Transparency a perceptual problem
and not only a physical one
2. The sensory problem (Hering - Helmholtz)
3. Why working on apparent instead
of real transparencies
~~E~~ models obtained by juxtaposing opaque surfaces
4. Conditions of transparency (ex Metzger's cross)
a) figural
b) color:

5. Type of research
not conformistic (at least for Psychology)
in general Collection of facts → curve fitting → algebr. exp.
→ theory
theory → algebraic expression → algebr.
manipulation → necessary conditions (if
theory is right → control on the facts

6. Influence of color in the perc. of transparency
I thing: quantitative expr. of colour
definition colour as stimulant 3 numbers
definition of shades of gray only one number
$L = \frac{i}{J}$ J intensity of light falling on a unit area

reason - achromatic colours defined by only one number
it appeared suitable to begin by confining
my study to the field of achr. colors

7. $\boxed{\Phi}$

~~every lig~~

Description of what happens ~~if~~ when we have the impression of transparency

P → two different perceptual effects: we see anterior layer T, which is transparent and ~~through~~ through it a second layer whose colour is the same as the colour of the contiguous region A. B

Perceptual scission one sort of stimulation
2 perc. effects (surface color and illumination)

8. Relation between stimulus color and scission color

Simple solution due to G. Hinder Moore and Koffka If stimulus color gray one scission colour blue the other has to be yellow

$$\cancel{G} = Y + B = G$$

than $G - B = Y$

Vedere come funziona q. ragionamento con le formule, tenendo conto di trasparenza e colore

Starting point of my research
Not algebraic formulation Y B G not symbolize numbers

~~Not~~

9 Possible to take the Law of color mixture to describe color scission

$$\overset{a}{} \quad \overset{b}{}$$

$$c = \frac{a+b}{2}$$

$$c = \frac{\overset{m}{m}a + \overset{n}{n}b}{m+n} = \frac{m}{m+n}a + \frac{n}{m+n}b$$

$$\frac{m}{m+n} + \frac{n}{m+n} = \frac{m+n}{m+n} = 1$$

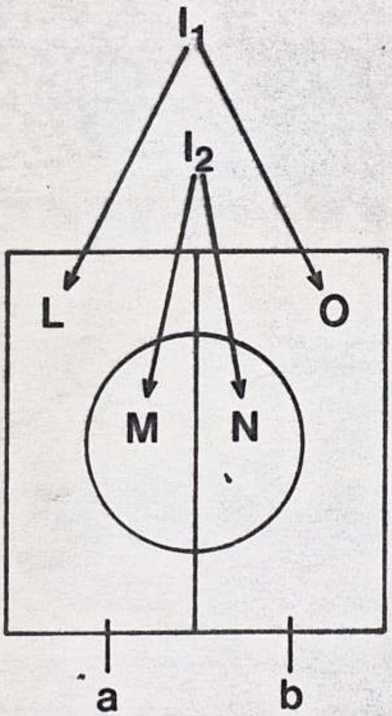


Fig. 1

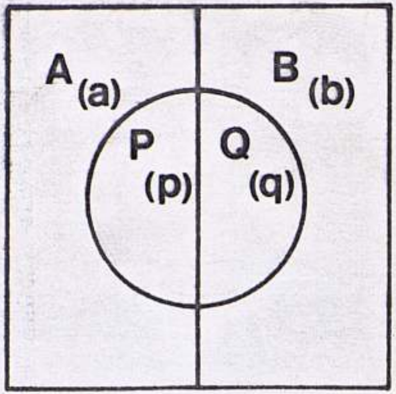


Fig. 2

guess = indovinare

Surface
Region
Layer

darker
brighter
bigger
smaller

Transparency

1 No perceptual problem
Experiment: 1) sheet of transparent plastic on homogeneous background
2) Metzger's Cross juxtaposition of pieces of opaque paper

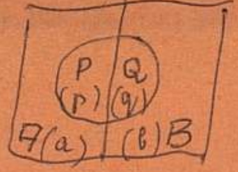
2 There are Necessary Conditions: { figural / colour } Explanation in terms of ones

3. Study of colour conditions Research following an unusual way (unorthodox)
[Scission theory] Starting from a theory → Expressing it in algebraic form → ~~manipu~~ solving the equation for the unknown elements → inferring necessary conditions → Controlling if facts correspond to inferences

3. Numerical expression of colour. For defining colours 3 numbers – extreme complication.
Achromatic colours (series Black – grey – white) only 1 number is needed.
Index of reflectance or albedo $L = \frac{i}{J}$ i – small, J – capital i = intensity of reflected light; J = intensity of light falling onto the area
Light reflected by surface divided by light falling on it
Absolute white reflects all = 1
Absolute black " nothing = 0
Various shades of grey 0 – 1
Therefore only achromatic colours will be used, and called simply colours

4. Model for transparency (similar to $\boxplus$; preferred because reproducible with episcotister)
Let us start with a simple figure – which can be considered a general model for perceptual transparency phenomena

regions capitals lett.
reflectances small lett.



Let us consider, for the moment, only one half of the model
Stimulation originated by P region produces 2 different perceptual effects: a transparent layer T and a layer seen through it, this one being of the same colour of the contiguous region A. The same for Q and B.

Problem: relation between stimulus colour and scission colours. (All perceivable in the figure).
[Which are the data, and which the unknowns? Solution seems to be very simple if we know the function relating p, a and t]

5. Simple ~~solution of~~ relation hypotized by G. Heider and K. Koffka (and proved, at least qualitatively) $t = f(p, a)$.
If stimulus colour gray and conditions require that one of the scission colours be blue, the other must be yellow
(If Y + B = Gray, then G – B = Y) Red + Green = Gray R + G = N
Gray – Green = Red N – G = R
not algebra. Starting point of my research

6. If the same law rules colour mixture and colour scission, we can use the ~~for~~ equation of colour mixture (Talbot's law) to describe ~~the~~ the phenomenon of colour scission, that is transparency. ~~(Heider and Koffka)~~
If two acromatic colours whose reflectances are a and b are mixed in equal quantities, the ~~mixture c~~ reflectance of the mixture c is the arithmetic mean of the reflectances of the two colours.

$c = \frac{a+b}{2}$ $c = \frac{1}{2}a + \frac{1}{2}b$ $c = \frac{1}{10}a + \frac{9}{10}b$

If a and b are mixed in different quantities, the reflectance of c is the weighted average of the components
$C = \alpha a + (1-\alpha) b$ Where α and $(1-\alpha)$ are the proportions in which the components are present in the mixture.

But if Talbot's law rules also colour scission, the same equation describes the phenomenon of colour scission, that is [using symbols according to the model

$$p = \alpha a + (1-\alpha) t$$

reflectance of
p = stimulus colours
a = reflectance of second layer
t = reflectance of transp. layer
$\alpha, 1-\alpha$ = proportions into which the stimulus colour has been divided into the 2 layers

7. What means α [proportion of colour going to 2 equal surfaces (2 equal glasses with water): more colour, less transparency, more visibility]

proportion of colour going to the second, opaque layer A.
As greater α, as more color goes to the second, opaque layer, the less colour goes to the first, transparent layer, as more the transparency

α = a coefficient of transparency (other coeff. = log. of the reflectance)

$p = \alpha a + (1-\alpha) t$

[if $\alpha = 0$ a disappears and $p = t$ we see only the layer t, which is therefore opaque
$(1-\alpha)=0$, t disappears from the equation
$\alpha = 1$ $p = a$ the layer T has no colour and disappears, and we see only a (perfect transparency)]

are ... [illegible]

In both cases, no colour scission.

8. With the interpretation of α ~~No each~~ Meaning of symbols in the equation clear: $\underline{a}$ and $\underline{t}$ = reflectances (colours) of the 2 scission layers.

But a is a known quantity (colour of the contiguous region A), while t is, together with α, one unknown quantity.

2 unknowns in the equation (α, t): solution indeterminate. [Relation between stimulus colour and scission colours we seems not able to state]

But so far we used only 1 half of the ABQB model

$$q = \alpha' b + (1-\alpha') t'$$

and if $\alpha' = \alpha$ and $t' = t$ we may put

The system of 2 equations in 2 unknowns is soluble

So we were able to predict not only the colour t of the transparent layer but also the degree of transparency

$$\alpha = \frac{p-q}{a-b} \qquad t = \frac{aq - bp}{(a+q)-(b+p)}$$

Solutions in terms of known quantities: albedos of the four surfaces a, p, q, b.

9. Checking the ~~form~~ deduced formulas; if and to what extent there is correspondence between theoretically deduced formulas and facts

Let us begin with Equation of the coefficient of transparency or more exactly phenomenal scission index $\alpha = \frac{p-q}{a-b}$ We can take the following way of reasoning.

Equation defining the ~~field~~ region of transparency: ($\alpha = 0$ perfect opacity $\alpha = 1$ perfect transparency are the limiting cases where phenomenal scission is lacking.
$\alpha > 1$ or $\alpha < 0$ → one layer would receive a negative quantity of colour, what is devoid of meaning. Therefore

$\alpha < 1$ 1. $|a-b| > |p-q|$ otherwise $\alpha \geq 1$

if $(a > b)$ then $(p > q)$
$\alpha > 0$ 2. if $(a < b)$ then $(p < q)$ otherwise $\alpha < 0$

Necessary conditions of transparency

Condition 1 If the difference in reflectance between the splitting regions P and Q is greater than the difference in reflectances between the non-splitting regions A and B, there cannot be transparency

$|p-q| < |a-b|$

$(\alpha < 1)$

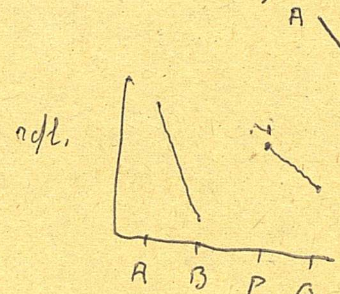
Ceck

in Fig. ~~[illegible]~~ 9 there is transparency: in fact the splitting regions are also in this case the regions between which there is less difference in reflectance. Condition 1 holds

Condition 2 the brightness fall (gradient) must have the same direction in the regions ~~A and B and P → Q~~

If $p > q$ then $a > b$

$(\alpha > 0)$



Ceck

<u>Prediction of transparency and opacity</u>

~~If~~ Looking at the formula we see that if the difference [in brightness (reflectance)] ~~[illegible]~~ between a and b is much greater than the difference in brightness between p and q, the numerator becomes little compared with the denominator, α is little, and the transparency is little; if the difference (in brightness) between p and q is hardly less than the difference (in brightness) between a and b, transparency is great.

<u>Intuitively</u>

$a > p > t$ or $t > p > a$

$b > q > t$ or $t > q > b$

$$\alpha = \frac{p-t}{a-t} \qquad \alpha > 0$$

if $\boxed{p > t \text{ then } a > t}$

Hyp. numerator and denominator positive

$$\frac{p-t}{a-t} < 1$$

$$\frac{(p-t)\cancel{(a-t)}}{\cancel{a-t}} < 1\,(a-t)$$

$$p - t < a - t$$

$\boxed{p < a} \rightarrow \boxed{a > p}$

~~[illegible]~~

$\boxed{\text{if } p < t \text{ than } a < t}$

numer. and den. negative

$$\frac{p-t}{a-t} < 1$$

$$\frac{(p-t)\cancel{(a-t)}}{a-t} > 1.(a-t)$$

$$p - t > a - t$$

$\boxed{p > a}$

$t > p > a$

$t > q > b$

<u>Algebraically deducted</u>

~~Predictions~~

$a > b > p > q > t$

$t > p > q > a > b$

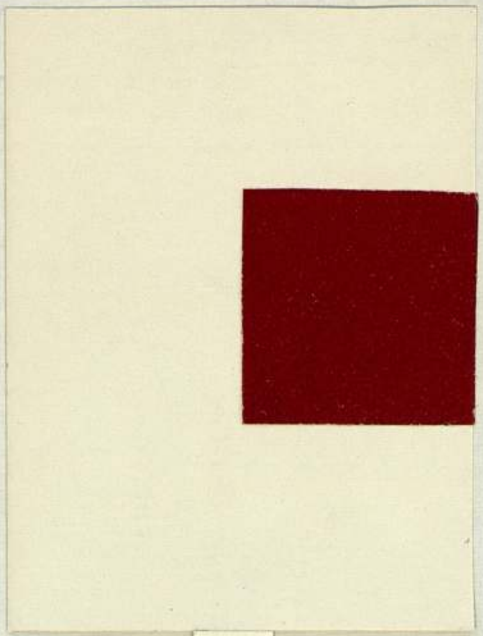
$a > p > t > q > b$

Following a way which is exactly the opposite of the usual one

usual order of operations ~~used~~: experiments → data → curve → curve-fitting equation

here: Theory → expressing with an equation → solving equation (giving the equation an appropriate form) ~~inferring~~ drawing inferences from the solution (if the equation ~~correct~~ expresses the phenomenon, then the phenomenon ~~has to follow certain depend~~ exists only if certain necessary conditions, inferred from the equation, were present) controlling if facts correspond to inferences.

2



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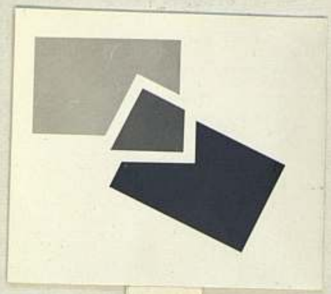

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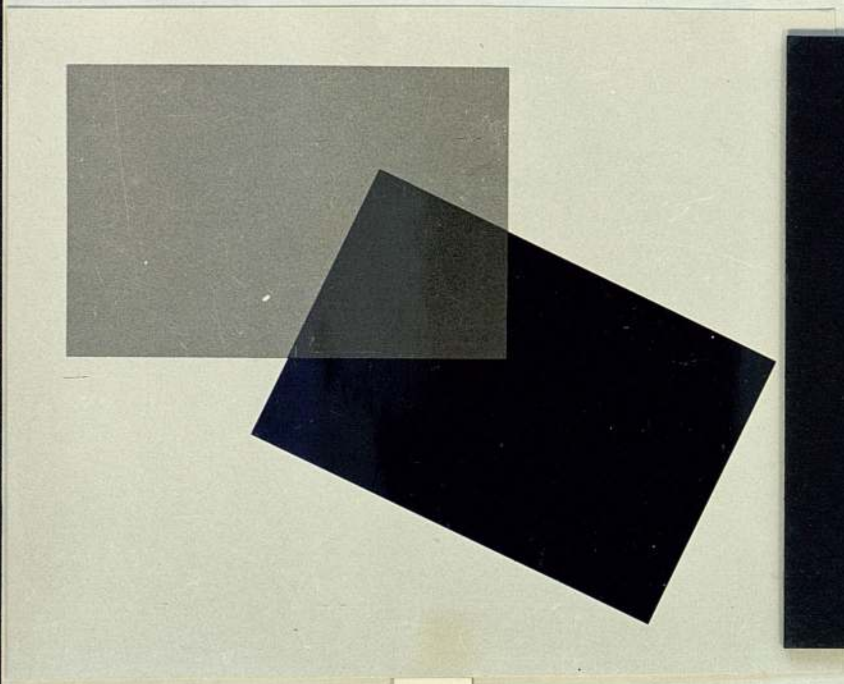

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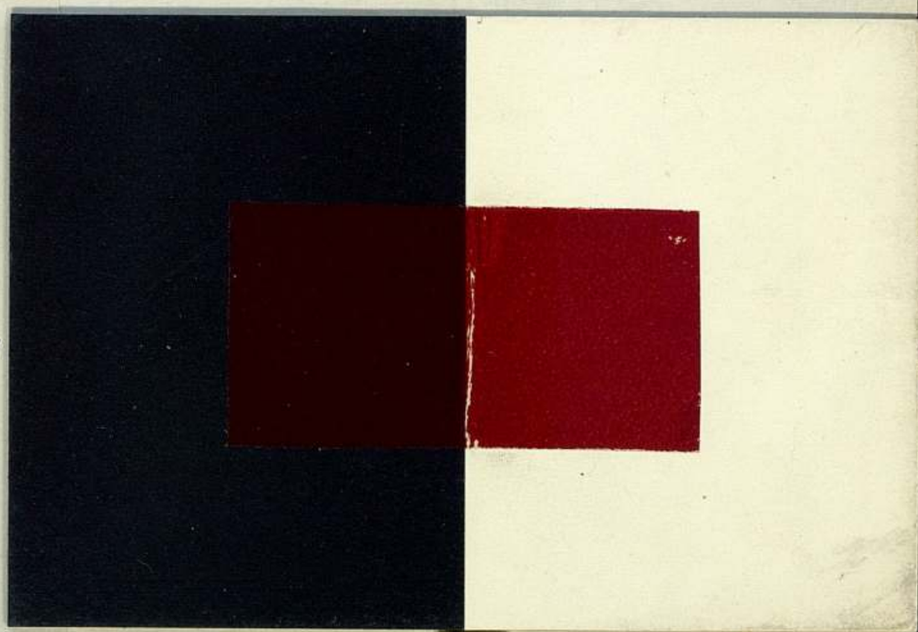

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horizontal liegend, die

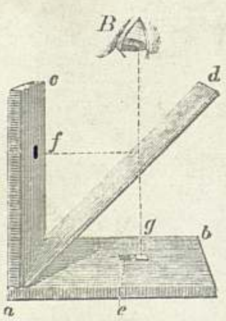

Fig. 151.

ässig kleinen Antheilen
nehrmals innerhalb der
unkel gefärbten Platten

from H. Helmholtz – Physiologische Optik

7

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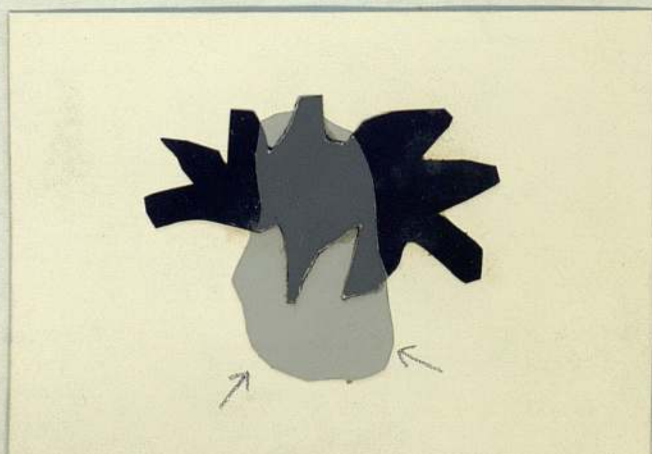

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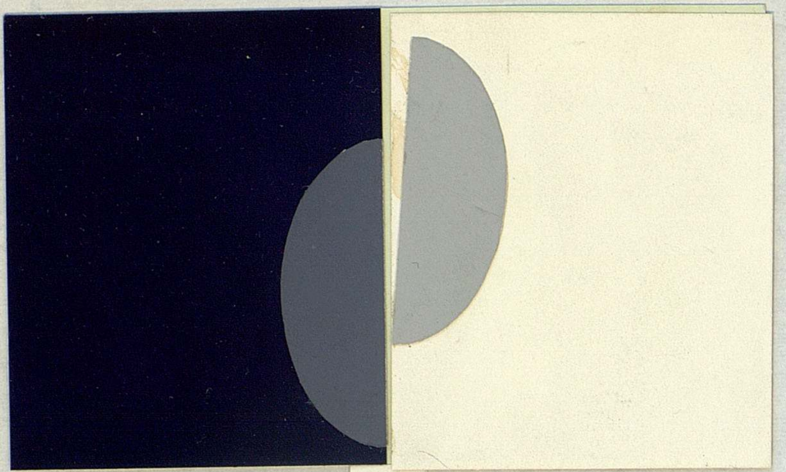

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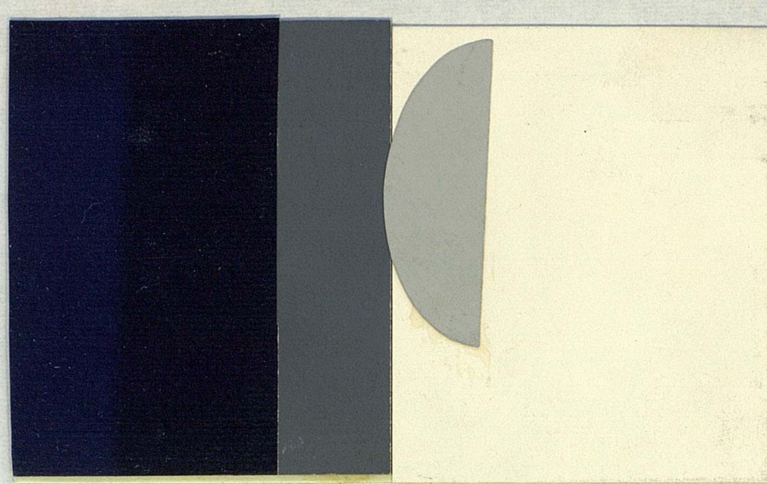

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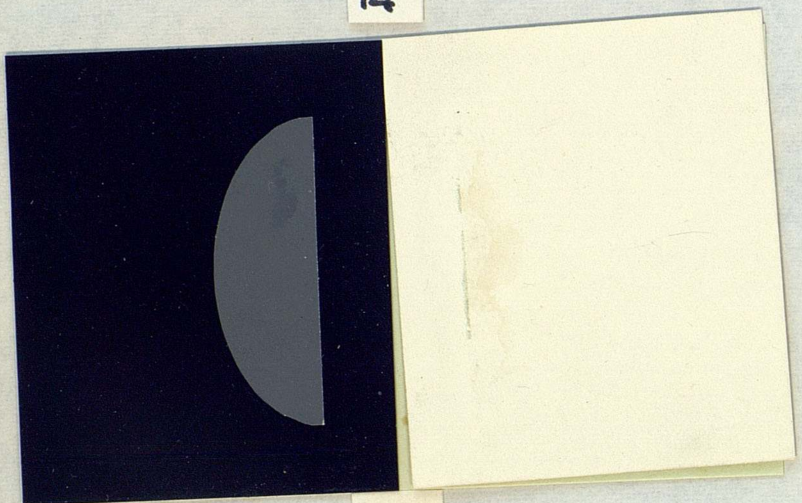

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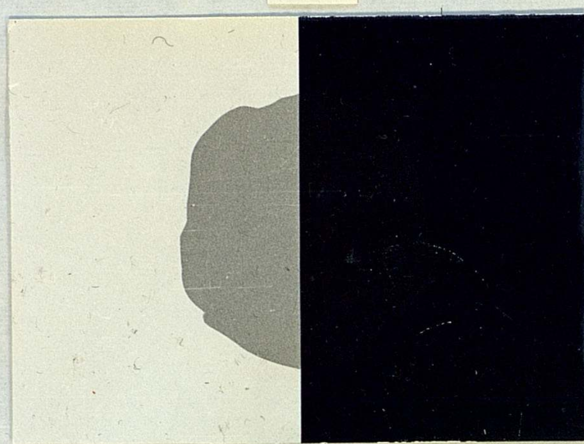

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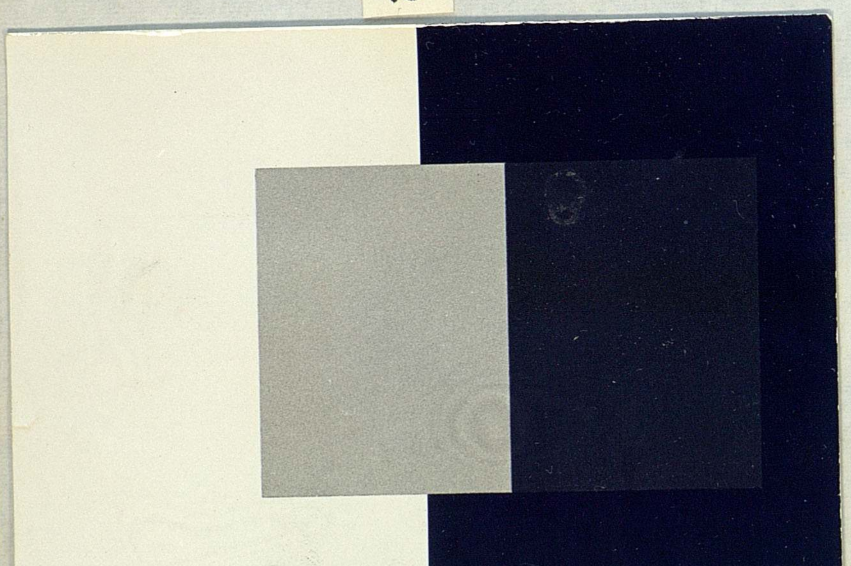

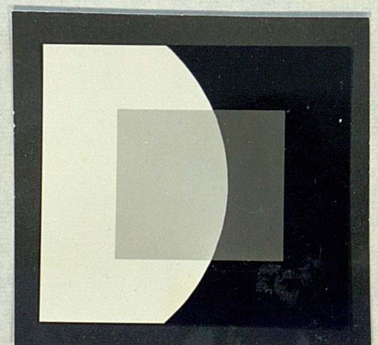

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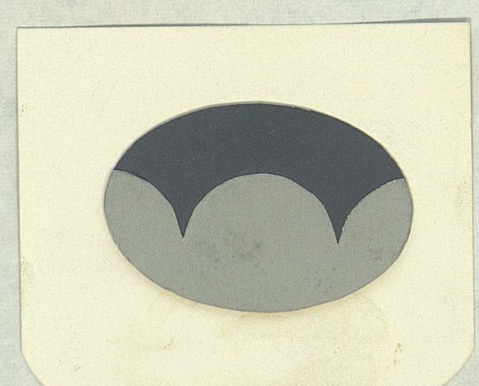

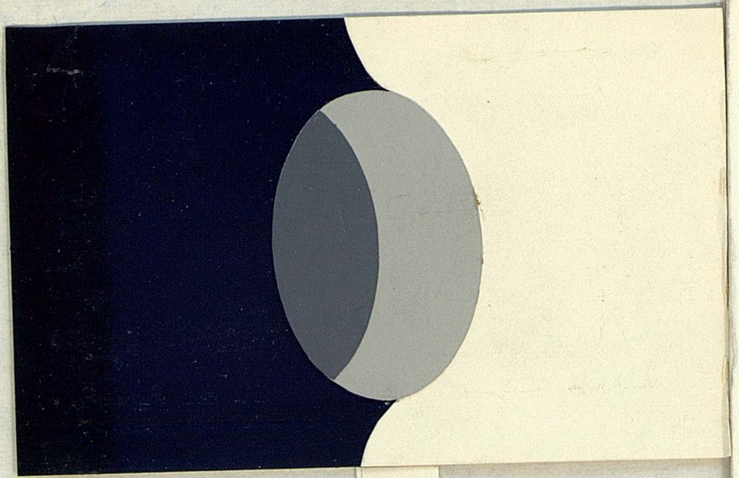

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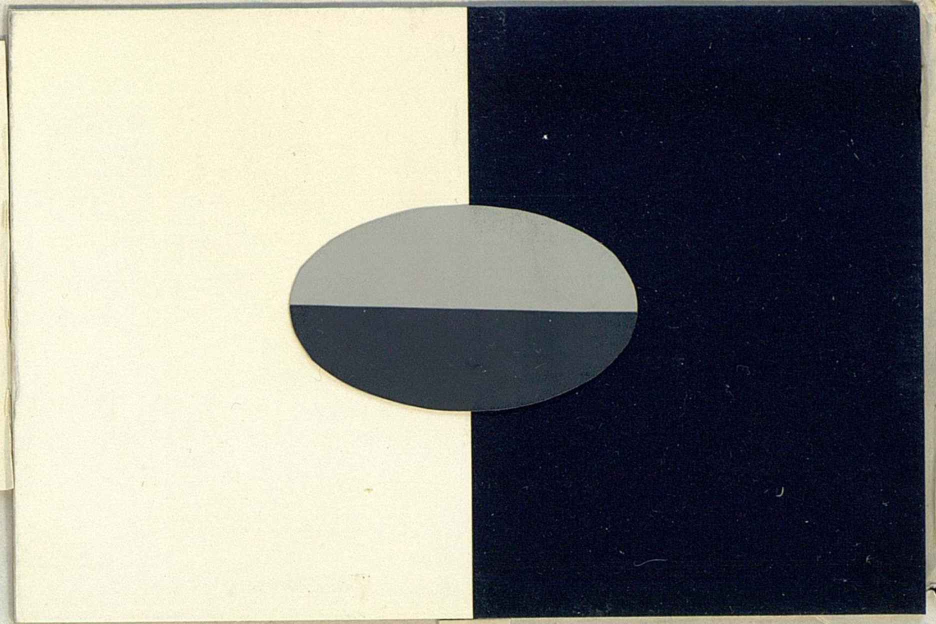

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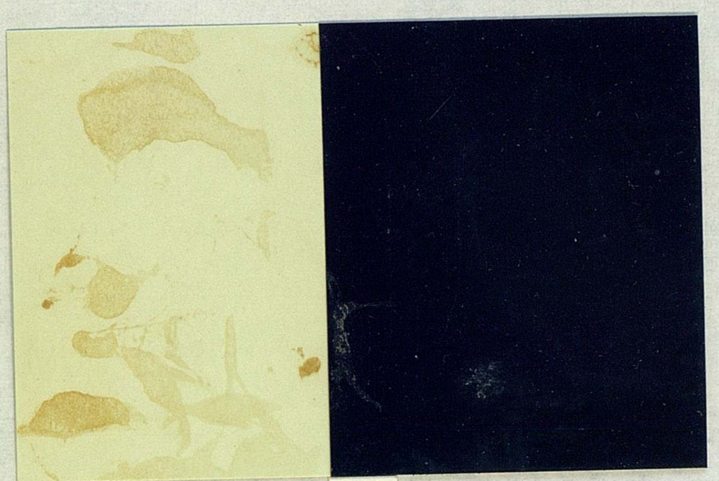

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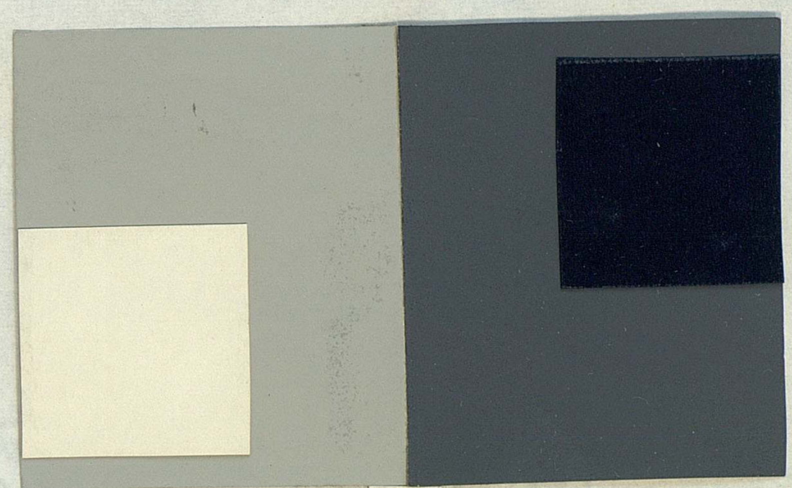

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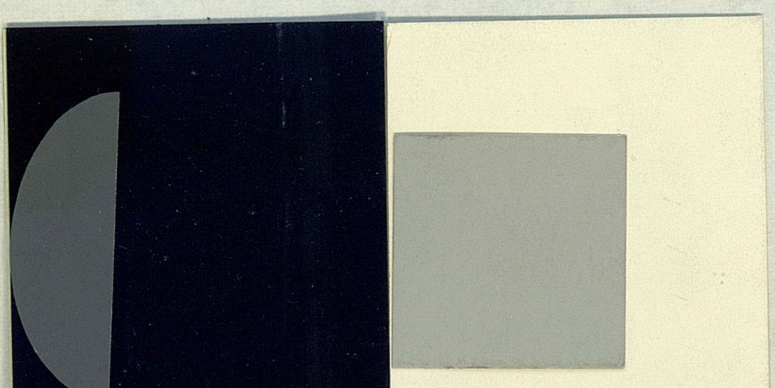

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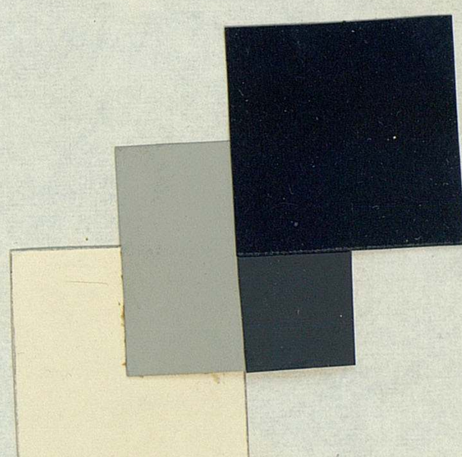

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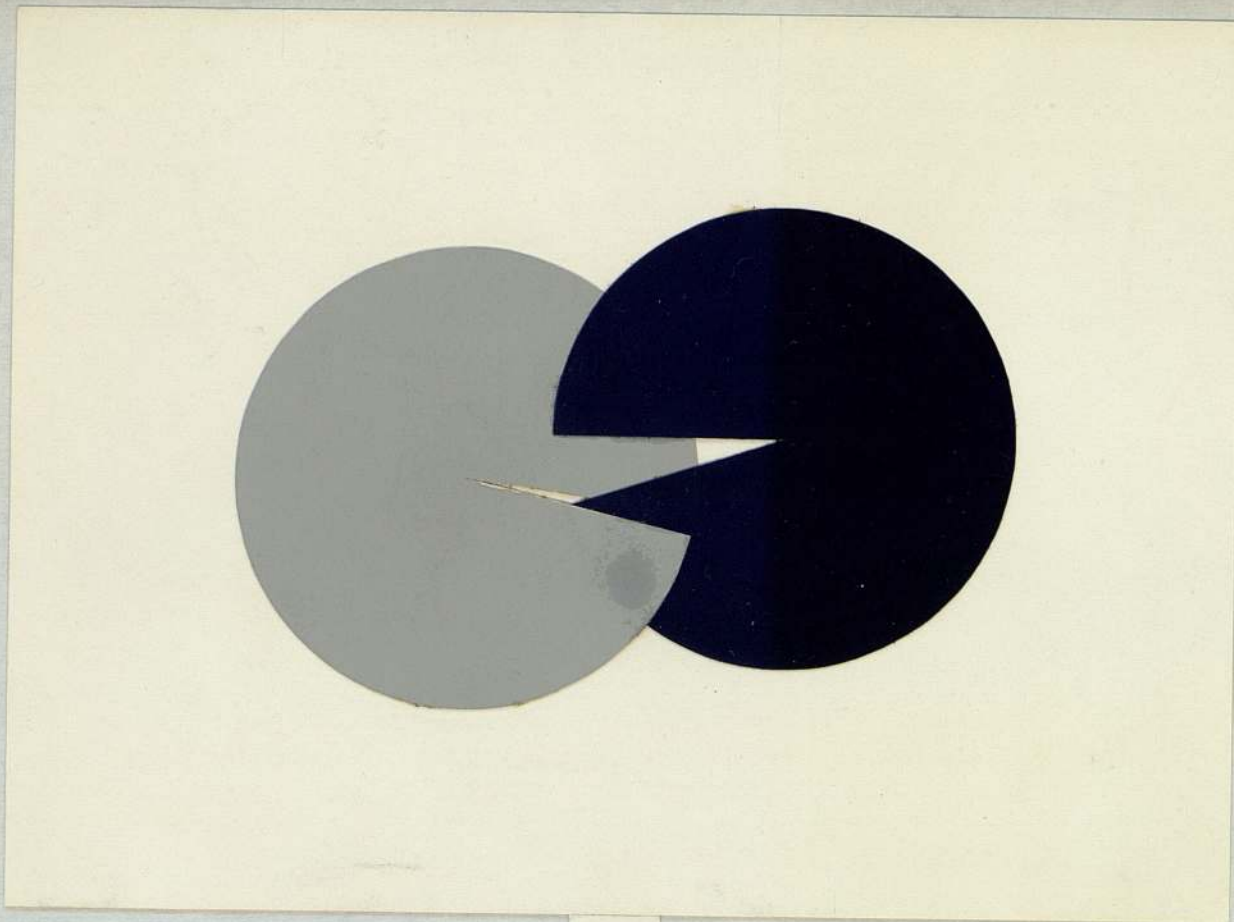

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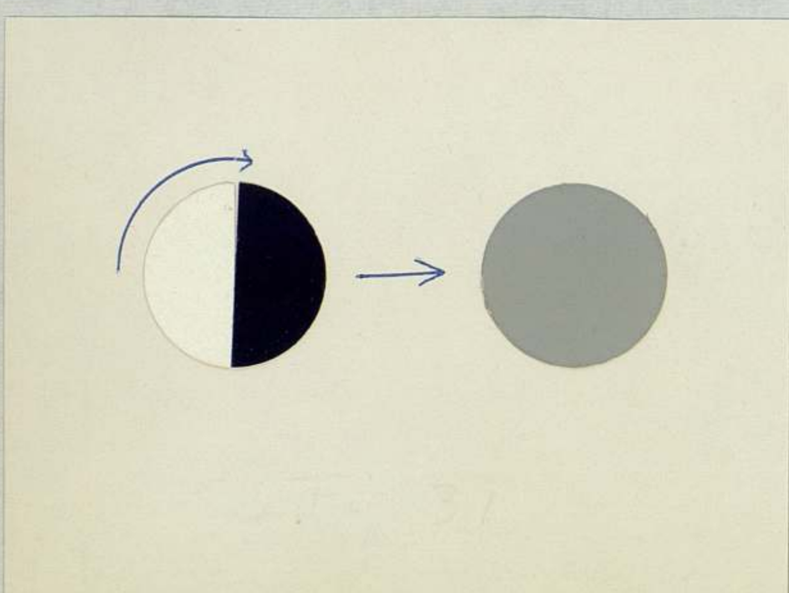

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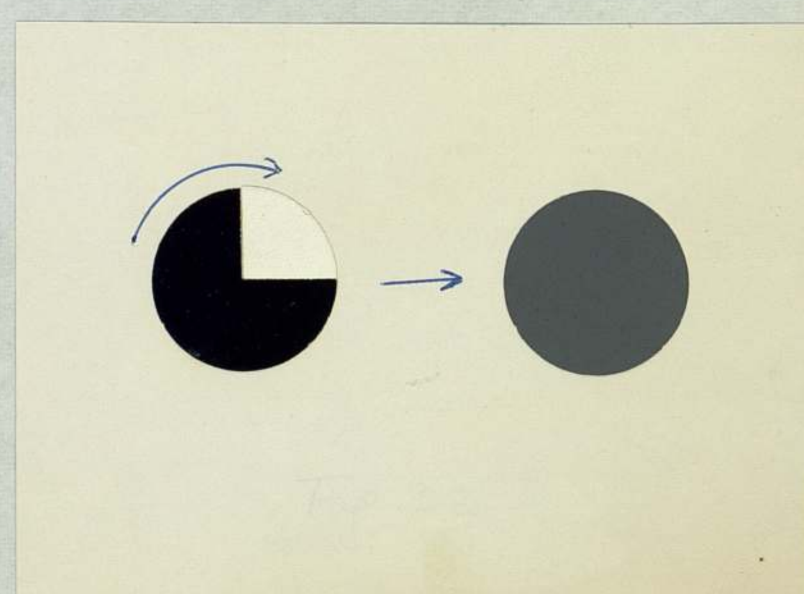

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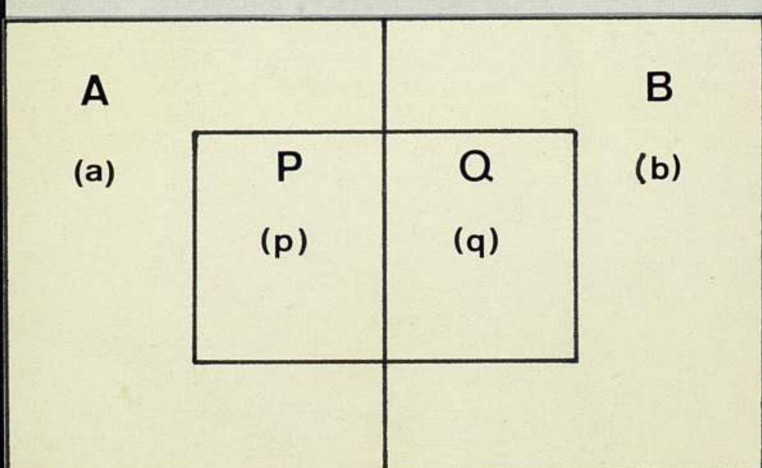
5.

Grey 1 : refl. = 21
Grey 2 : refl. = 53

Fusion color : refl. $= \frac{1}{4}\,21 + \frac{3}{4}\,53 = 45$



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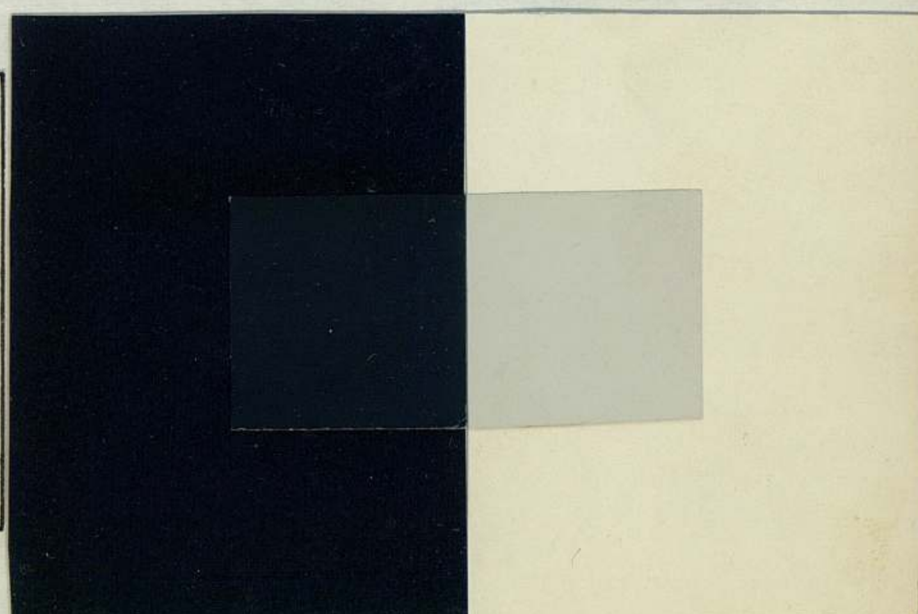

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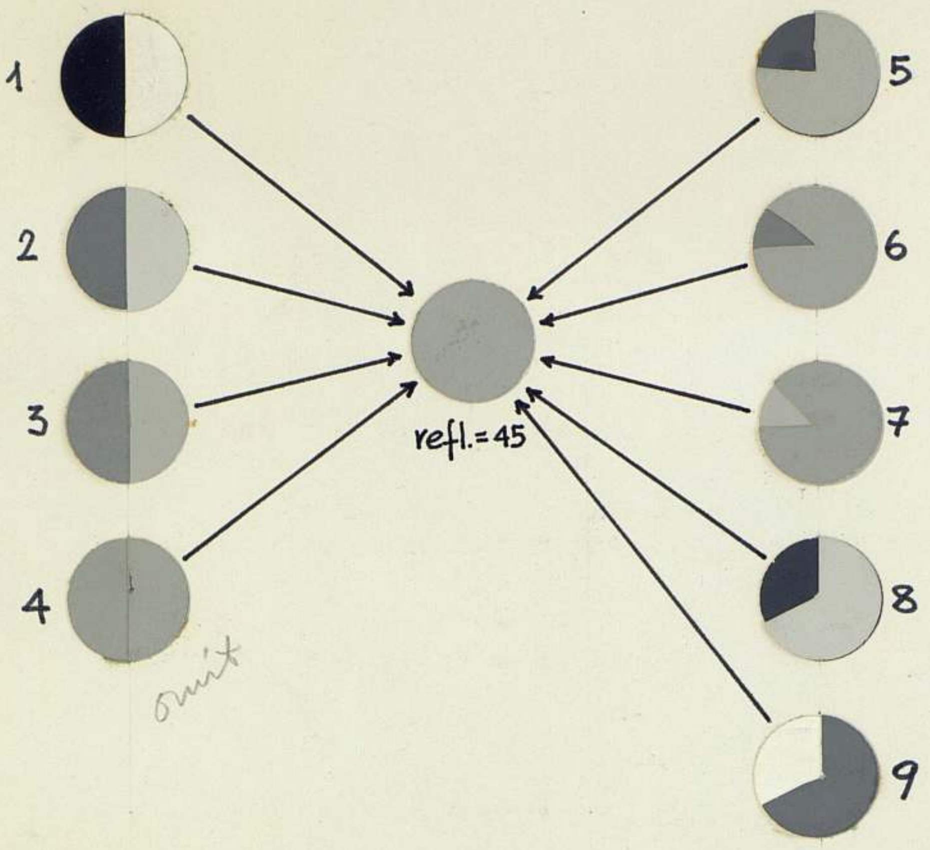


33

1.
Black : refl. = 4
White : refl. = 86

Fusion color: refl. = $\frac{4 + 86}{2}$ = 45

2.
Grey 1 : refl. = 26
Grey 2 : refl. = 64

Fusion color: refl. = $\frac{26 + 64}{2}$ = 45

3. Grey 1 : refl. = 53
Grey 2 : refl. 37

Fusion color : refl. = $\frac{33 + 53}{2}$ = 45

4.
Grey 1 : refl. = 43
Grey 2 : refl. = 47

Fusion color : refl. = $\frac{43 + 47}{2}$ = 45

5.
Grey 1 : refl. = 21
Grey 2 : refl. = 53

Fusion color : refl. = $\frac{1}{4}$ 21 + $\frac{3}{4}$ 53 = 45

6.
Grey 1 : refl. = 31
Grey 2 : refl. = 47

Fusion color : refl. = $\frac{1}{8}$ 31 + $\frac{7}{8}$ 47 = 45

7.
Grey 1 : refl. = 55.5
Grey 2 : refl. = 43,5

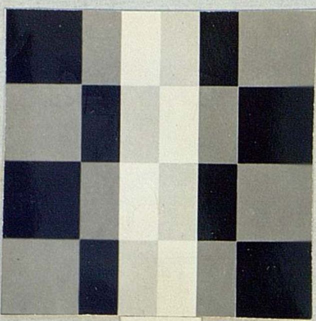
Fusion color : refl. = $\frac{1}{8}$ 55,5 + $\frac{7}{8}$ 43,5 = 45

8.
Grey 1 : refl. = 9
Grey 2 : refl. = 63

Fusion color : refl. = $\frac{1}{3}$ 9 + $\frac{2}{3}$ 63 = 45

9.
White : refl. = 86
Grey : refl. = 24,5

Fusion color : refl. = $\frac{1}{3}$ 86 + $\frac{2}{3}$ 24,5 = 45



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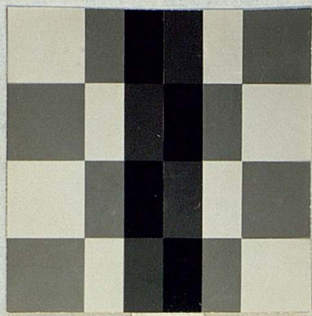

43

1 2

4 7

p = a
q = b

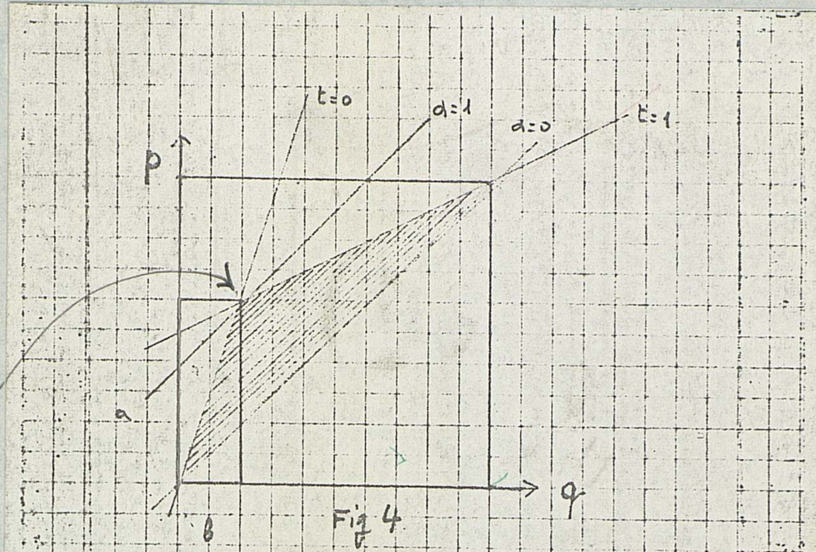

Fig 4

The region of perceptual transparency
($0<\alpha<1$ and $0<t<1$)

44

see also 20-21
Figs 23-27 + 34
in orig mss

3 4