I cerchi grandi sono i ~~colori~~ punti corrispondenti ad un dato quadrato di data riflettanza — senza filtro con la stima corrispondente

I ~~colo~~ cerchi piccoli — con filtro —

bianchi — sfondo chiaro

neri — sfondo scuro

Fondamentale il fatto che si dispongono tutti lungo una stessa curva

Più il filtro è opaco più sono schiacciati

verso l'ascissa perché più scuri

Le equazioni non vanno calcolate

Questi sono grafici che ~~no~~ ci hanno permesso

non disponendo dei dati per elaborare la legge di Talbot

La figura è poco chiara perché non c'è il

Dato della trasmittanza

MANUSCRIPT SUBMITTED TO PERCEPTION, 1982

Osservare il testo di Bourdon e della Heider

A psychophysical study of Fuchs phenomenon

Sergio Cesare Masin

Department of Psychology, University of Padua, Italy

Received 2 December 1982, in revised form 1983

Perché non riporre gli esperimenti di Koffka e Bourdon con filtri colorati: la costante tra filtro e figura fu ridotta a zero.

ABSTRACT. The Fuchs phenomenon (i.e., the perception of transparency when the object seen through transparency lies on a homogeneous background, and does not jut out under the transparent layer) is studied here by the numerical rating method in two experiments, with the use of achromatic colours. The results of the first experiment show that the degree of transparency depends on the difference in colour between the object and the background. The results of the second experiment suggest that the processes producing the phenomenon are different in nature from the processes producing the kind of transparency most frequently studied, in which the object juts out under the transparent layer.

1 INTRODUCTION

If we look at an object lying on a chromatically homogeneous background, through a chromatically homogeneous filter, so that the background is so wide that its contour entirely includes the contour of the filter, and the filter is so wide that its contour entirely includes the contour of the object (case 1 in figure 1, where the proximal stimulation is depicted) then the filter is perceived as transparent both on the background and on the object. If the object is displaced on the background so that it now juts out under the filter (case 2 in figure 1), the filter is still perceived as transparent. If the object is further displaced so that now it is entirely out of the filter (case 3 in figure 1), the filter is compellingly perceived as opaque. (See Fuchs, 1923.)

This kind of transparency in which only three proximal surfaces are involved and no contour is intersecting (case 1 in figure 1), is termed here as the Fuchs phenomenon. So far, experimental results have been obtained primarily in studies of transparency using patterns producing four proximal surfaces with intersecting contours (case 2 in figure 1; Metelli, 1970, 1974 a, 1974 b, 1975; Kanizsa, 1980; Metelli, Masin, and Manganelli, 1981), and patterns producing only two proximal surfaces (case 3 in figure 1; Masin and Idone, 1981). The Fuchs phenomenon has never been studied quantitatively. The two experiments reported in the following sections had the

e la tr. parziale?

=====================
Figure 1 about here
=====================

purpose to quantitatively explore this phenomenon. The first experiment was aimed at detecting the chromatic relations that affect the degree of perceived transparency. ?
The second experiment served to check on Koffka's (1935) qualitative explanation of the Fuchs phenomenon.

? Cosa intendi? Rapporti fra colori e trasparenza?

2 EXPERIMENT 1

2.1 METHOD

2.1.1 Subjects

There were twenty subjects. They were students and personnel at the Psychology Department.

2.1.2 Stimuli

The subject sat on a chair, and a chin rest was used to keep the eyes level with the stimulus. Two exactly superimposed glass sheets of 1.5 x 220 x 275 mm were put in a rectangular wooden frame, 2.5 m distant from the subject's eyes. Behind the subject there was a homogeneous black curtain (reflectance about .05). The wooden frame was first placed on a plane frontal-parallel to the subject, then slightly tilted of about 7 deg backwards so that the glasses in the frame reflected only the uniform black curtain just above the subject's head. Thus, from subject's standpoint no reflected image could be detected in the glasses. Six frames, with two glass sheets in each of them, were built. A (Kodak Wratten No 96) achromatic gelatine filter was placed in between the two glasses of a frame so that the filter appeared as suspended in space in the middle of the frame. A given achromatic filter, of 72 x 76 mm, had one of the following transmittances: .10, .15, .25, .40, .65, or .80. At 2.9 m from subject's eyes, an achromatic background cardboard of 210 x 250 mm, tilted 7 deg backwards and parallel to the frame, was placed exactly behind

4

the frame. A small achromatic square of 23 x 23 mm was stuck in the middle of the background. The position of the square on the background was such that the subject perceived it in the middle of the filter (as in the case 1 depicted in figure 1). The subject viewed the scene binocularly. Ten backgrounds, with a square on it, were constructed. Half the backgrounds had a reflectance of .16, and the other half had a reflectance of .60. The reflectance of a square on a background of these two sets of backgrounds was .04, .11, .25, .45, or .75 (Hesselgren papers). The light came from neon tubes on the ceiling. The illumination level was of 60 lx.

2.1.3 Procedure

The subjects were shown at random all the sixty combinations of the filters and squares on the background. A given combination was presented once. For each combination, the subjects were asked to assign a number from 1 to 9 to each filter according to the degree of perceived transparency of the filter (the higher the degree, the greater the number). Before starting the experiment, the subjects were shown an example of a very high degree of transparency, and one of a very low degree. A session lasted about half an hour.

3 RESULTS AND DISCUSSION

The results are reported in tables 1 and 2, as the means of 20 subjective estimates of the degree of apparent transparency, for all combinations of the transmittances

5/

of the filter and the reflectances of the square seen through transparency. Table 1 refers to the dark background having a reflectance of .16, and table 2 to the light background having a reflectance of .60.

==========================
Tables 1 and 2 about here
==========================

In figure 2, the estimated degree of transparency is plotted as a function of the reflectance of the square, for the dark (left) and light (right) background. The curves, drawn up by eye, fit the data points corresponding to the indicated transmittances of the filter. The standard error of a mean estimate varies between .2 and 1.2 the size of a dot. All subjects showed essentially the same pattern of results. (Notice that the scale of measurement for the apparent degree of transparency obtained here is ordinal.)

?

Perchè?

====================
Figure 2 about here
====================

As may be seen, the perceived degree of transparency of the phenomenal filter diminishes as the difference in colour between the square and its background diminishes. Since the filter is perceived as opaque when this difference is null (Fuchs, 1923), it is assumed that, by extrapolation, the curves pass through the point of origin (.16, 0) for the diagram on the left, and (.60, 0) for the diagram on the right.

We must now ask why phenomenal transparency occurs in the Fuchs phenomenon for a given difference in colour between the square and its background. Koffka (1935, p. 261), the only author who dealt with a theoretical interpretation of the Fuchs phenomenon, tried to explain it as follows. Let a blue-coloured episcotister revolve at fusion speed in front of a square having a complementary colour. If the square is stuck on a black background, a Fuchs phenomenon is generated so that a blue disk-shaped phenomenal surface (generated by the episcotister) is perceived as transparent in front of a yellow square against a black background. However, if we look at the square through a reduction screen, we perceive a grey colour, which is the result of the mixing of the blue and the yellow. So, where does the yellow of the square come from, when the scene is looked at without a reduction screen?

According to Koffka, the yellow is the result of a process that integrates the chromatic information coming from the parts of the retina (or better, of higher stations) that correspond to the different phenomenal surfaces. The result of this integration process would be that the reduction colour of the phenomenal square (i.e., the colour of the square when, under the same stimulus conditions, the phenomenal transparency is abolished by an analytical attitude. This is also the colour of the square seen through a reduction screen, under the same chromatic contrast conditions) is split, so to speak, into two complementary colours. But what colours? The answer depends on which is the colour of the part of the transparent layer covering the background. In Koffka's example, the episcotister is blue, and consequently the colours which are split are the yellow and the blue.

Bourdon (1936) made observations suggesting that Koffka was wrong. He used an experimental setting identical to the one described by Koffka. In Koffka's example the Fuchs phenomenon occurs because the depth conditions (mainly retinal disparity) displace the transparent layer (the phenomenal disk generated by the episcotister) forward in the third phenomenal dimension. If these conditions are not made to act, then the Fuchs phenomenon may be abolished. chi lo dice? This Bourdon did by reducing to very, very few millimetres the physical distance between the episcotister and the square on the background. By viewing this scene at a suitable distance, transparency was suppressed. No In this new situation, the square appeared as being yellowish as it was before when it was perceived through transparency. According to Bourdon, this would show that chromatic contrast is responsible for the shade of yellow in the phenomenal square perceived through transparency in Koffka's example.

In point of fact, Koffka borrowed the hypothesis of the splitting of colours from Moore-Heider (1933) who studied the kind of transparency illustrated as the case 2 in figure 1. Clearly, for Koffka this form of transparency and the Fuchs phenomenon were implicitly assumed to be ruled by the same basic process of splitting. Bourdon's observation suggests that Koffka's hypothesis of splitting does No not hold as regards the Fuchs phenomenon.

Bourdon reported only personal phenomenological observations, and did not collect quantitative data. Adopting a different procedure, the following experiment serves as a quantitative check on Koffka's explanation of the Fuchs phenomenon in terms of a process of splitting, when achromatic colours are used.

4 EXPERIMENT 2

4.1 METHOD

4.1.1 Subjects

There were ten subjects. They were students and personnel at the Psychology Department, different from those used in the experiment 1.

4.1.2 Stimuli

The stimuli were the same as in the experiment 1, except for the transmittances of the filters, that now were .10, .20, .40, .65, .80, and 1.00 (that is, a wooden frame with two glass sheets and no filter).

4.1.3 Procedure

The subjects were shown at random all the sixty combinations of filters, squares, and backgrounds. A given combination was presented once. For each combination, the subjects were saked to assign a number from ~~1~~ 0 to ~~9~~ 10 to the grey colour of the square on the background behind the filter (the lighter the grey, the larger the number). Before starting the experiment, the subjects were shown an example of a very dark grey, and one of a very light grey. A session lasted about half an hour.

5 RESULTS AND DISCUSSION

The results are reported in tables 3 and 4, as the means of 10 subjective estimates of the greyness of the square seen through the filter, for all combinations of the transmittances of the filters and the reflectances of the squares. Table 3 refers to the dark background (reflectance .16), and table 4 to the light background (reflectance .60).

===========================
Tables 3 and 4 about here
===========================

In figure 3, the estimated greyness is plotted as a function of the transmittance of the filters, for the dark (left) and light (right) background. The hyperbolic arcs, some drawn up by eye and some using the least squares method, fit the data points corresponding to the indicated reflectances of the square on the background. The standard error of a mean estimate of greyness varies between .2 and 1.2 times the size of a dot. All subjects showed essentially the same pattern of results. (Notice that the scale for greyness is ordinal.) ?

=====================
Figure 3 about here
=====================

Koffka's assertion that a yellow figure is still perceived as yellow when it is looked at through a rotating episcotister having a complementary blue colour, may be rephrased saying that the perceptual mechanism operates, in this instance, as if it were "regressing to the real object" on the background. That is, the colour of the figure perceived through transparency would

No, Koffka lo nega esplicitamente

be the same as that of the figure when there is no filter. no
As one can see in figure 3, the greyness of a square of a given reflectance seen through transparency tends to a common value as the transmittance of the filter approaches zero. (The curves in the diagram on the left are displaced upwards, with respect to the corresponding curves in the diagram on the right. This effect may be attributed to the chromatic contrast, as the curves on the left correspond to squares surrounded by a darker area.) This result shows beyond doubt that the grey colour of a square seen through transparency is never the grey colour that the same square has when there is no filter. (If a "regression to the real object" were somehow occuring, straight lines parallel to the abscissa would result in place of the hyperbolic arcs depicted in figure 3; a broken segment of these lines is indicated in the diagram on the right in figure 3.)

Then, what does the colour of the square seen through transparency correspond to? The big dots in figure 4 represent estimates of the points of the psychophysical function relating greyness to the reflectance of the square, for a transmittance of 1.00 (that is, when there is no filter). The filled and unfilled dots correspond to the dark and light background respectively. Using a measured value of
? .05 for the reflectance of the filter, and considering that

====================
Figure 4 about here
====================

the intensities of the light falling on the filter and on the ?
square were practically tha same when photometrically compared, the reflectance that a square-shaped piece of grey paper

in front of the filter, and subtending the same visual angle as the square, must have to produce the same proximal stimulation as that produced by the square behind the filter, was calculated for each square. The smaller dots in figure 4 represent the plots of the estimated greyness of the squares seen through transparency against the reflectance so calculated. The filled and unfilled dots correspond to the dark and light background respectively. The filled dots are slightly displaced upwards owing to chromatic contrast.

?

a che tabella ti riferisci

As may be seen, the big dots lie in the middle of the cloud of small dots. This shows that the subjects estimated numerically the reduction colour of the square (equal to the colour of the virtual square-shaped piece of paper supposed to be in front of the filter) which, however, they perceived as pertaining to the square behind the filter. This is in accordance with Bourdon's observations and contrary to Koffka's hypothesis as applied to the Fuchs phenomenon. This conclusion is confirmed by the following remark. From figure 4, a reflectance of .05 (the reflectance of the filter) corresponds to an estimated greyness of about 1.5. In figure 3, the hyperbolic arcs approximately converge in the point having the ordinate of 1.5 (this is clearer in the diagram on the right). This ultimately shows that the estimated colour of the square behind the filter tends to become the (reduction) colour of the filter as the transmittance approaches zero.

Benussi?

Questo sembra ovvio

6 CONCLUSION

Let us consider the kind of transparency illustrated in figure 1.2. When an analytical attitude is assumed, the colour of the parts of the object corresponding to the proximal regions X and Y appears as being different in the two parts. However, if a realistic or naive attitude is assumed (as commonly happens in everyday life), the colour of the part of the object seen directly is perceived as being identical to the colour of the part of the object appearing behind the transparent layer (Fuchs, 1923).

non è esatto. C'è nuovo colore (Metelli)

According to the theoretical and experimental results by Metelli (1970, 1974 a, 1974 b, 1975), the reduction colour corresponding to the region Y is in between the colour of the object seen directly and the colour of the transparent layer. This amounts to saying that the (reduction) colour of the analytically-perceived part corresponding to Y splits into two colours.

Non mi pare che la scissione derivi da questo ragionamento

It may reliably be stated that, since the object seen through transparency in the Fuchs phenomenon (figure 1.1) is not composed of analytically-perceived parts (so that the colour of one of these is used as a reference colour to be rendered identical to the colour of any other part, by the perceptual mechanism under the realistic attitude), the splitting of the reduction colour of the object is not possible.

?

No

In fact, this is what has been found in the second experiment. This strengthens the conclusion that the Fuchs phenomenon and the kind of transparency depicted in figure 1.2 are different both as to the topological relation among the different regions, and as to the perceptual processes producing the perception of transparency in the two cases.

This conclusion is of obvious theoretical importance since it implies that Metelli's theory (1970, 1974 a, 1974 b, 1975), in which the notion of a process of splitting has been successfully used to explain the kind of transparency depicted in figure 1.2, is not applicable to the Fuchs phenomenon.

The results of the first experiment have shown that the degree of apparent transparency is dependent on the difference between <u>the reduction colour</u> of the object and that of its background. The fact that this difference has a functional influence on the degree of perceived transparency does not mean, of course, that it is the only condition for the generation of the Fuchs phenomenon. At the present stage of research, the question why transparency occurs, for a given difference in colour between the object and its background, still remains unanswered.

ACKNOWLEDGMENT. I carried out some tentative investigations about the Fuchs phenomenon when, in the summer of 1980, I was a Fulbright student associated with N Pastore at Queens College, New York. I wish to thank
for careful reading of the first draft of the manuscript, and for useful discussion. I also wish to thank RA Salvà for assistence in running the subjects of these experiments.

REFERENCES

Bourdon B, 1936 "Sur une expérience de transparance" <u>Journal de Psychologie</u> 33 712-716

Fuchs W, 1923 "Experimentelle Untersuchungen über das simultane Hintereinandersehen auf derselben Schrictung" <u>Zeitschrift für Psychologie</u> 91 145-235

Kanizsa G, 1980 "Grammatica del vedere" Bologna, Mulino

Koffka K, 1935 "Principles of Gestalt psychology" New York, Harcourt, Brace

Masin S C, Idone A M, 1981 "Studio sperimentale sulla percezione della trasparenza con figura e sfondo acromatici e omogenei" Giornale Italiano di Psicologia 8 265-277

Metelli F, 1970 "An algebraic development of the theory of perceptual transparency" Ergonomics 13 59-66

Metelli F, 1974 a "The perception of transparency" Scientific American 230 (4) 90-98

Metelli F, 1974 b "Achromatic color conditions in the perception of transparency" in Perception, eds Macleod R B and Pick H L (Ithaca, New York, Cornell Univ Press), p 95-116

Metelli F, 1975 "On the visual perception of transparency" In Studies in Perception ed D'Arcais G B F (Milano, Martello), p 445-487

Metelli F, Masin S C, Manganelli A M, 1981 "Partial transparency" Atti e Memorie dell'Accademia Patavina di Scienze, Lettere ed Arti 92 115-169

Moore-Heider G, 1933 "New studies in transparency, form, and colour" Psychologische Forschung 17 13-55

Reflectance	Transmittance .10	.15	.25	.40	.65	.80
.04	1.7	2.9	4.2	5.1	6.5	8.2
.11	1.3	1.3	2.0	3.9	5.4	6.3
.25	1.3	2.1	2.6	4.1	5.3	6.3
.45	2.1	3.4	4.1	5.4	6.7	8.1
.75	3.6	4.6	5.2	5.8	7.3	8.2

Table 1

Reflectance	Transmittance .10	.15	.25	.40	.65	.80
.04	3.6	5.0	6.0	6.9	7.8	8.7
.11	3.6	4.6	5.5	6.3	8.0	8.5
.25	2.8	3.8	4.1	6.1	7.2	8.2
.45	1.7	2.1	3.3	4.8	5.9	7.5
.75	1.6	2.2	3.6	5.2	6.6	7.8

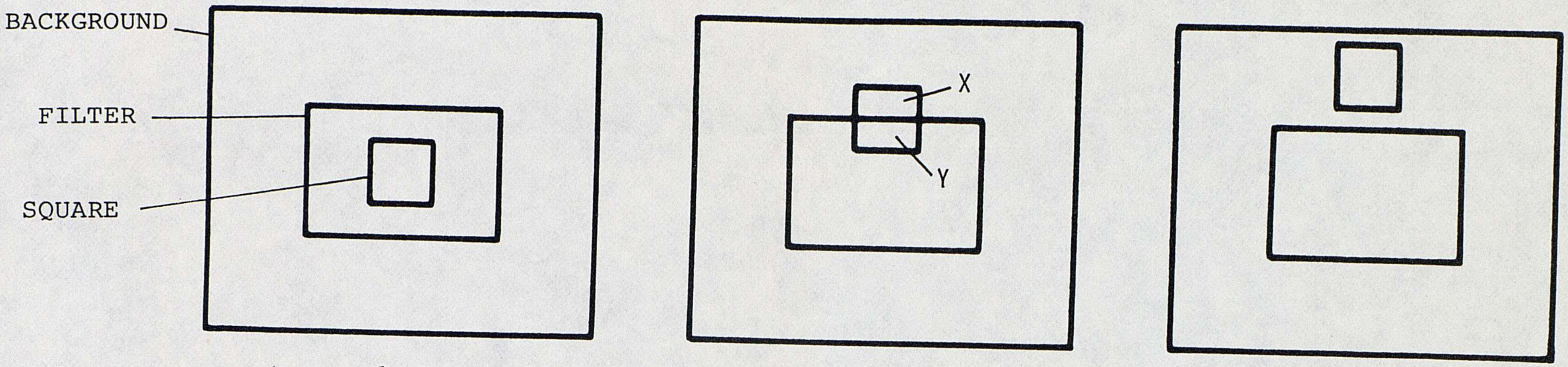
Table 2

Reflectance	Transmittance .10	.20	.40	.65	.80	1.00
.04	0.9	1.8	1.5	2.1	1.3	1.0
.11	1.0	2.1	2.6	3.7	3.9	4.4
.25	4.1	5.1	5.6	6.3	6.6	6.7
.45	5.7	6.6	6.6	7.2	7.6	8.1
.75	6.1	6.5	7.3	8.5	8.8	9.5

Table 3

Reflectance	Transmittance .10	.20	.40	.65	.80	1.00
.04	1.4	1.4	1.3	1.4	0.8	0.9
.11	1.5	1.7	1.8	2.0	2.4	2.9
.25	2.4	3.0	4.2	4.8	4.9	5.9
.45	2.6	4.4	6.2	6.6	7.3	7.9
.75	4.1	6.1	7.4	8.6	9.3	9.7

Table 4



1. 2. 3.

Figure 1

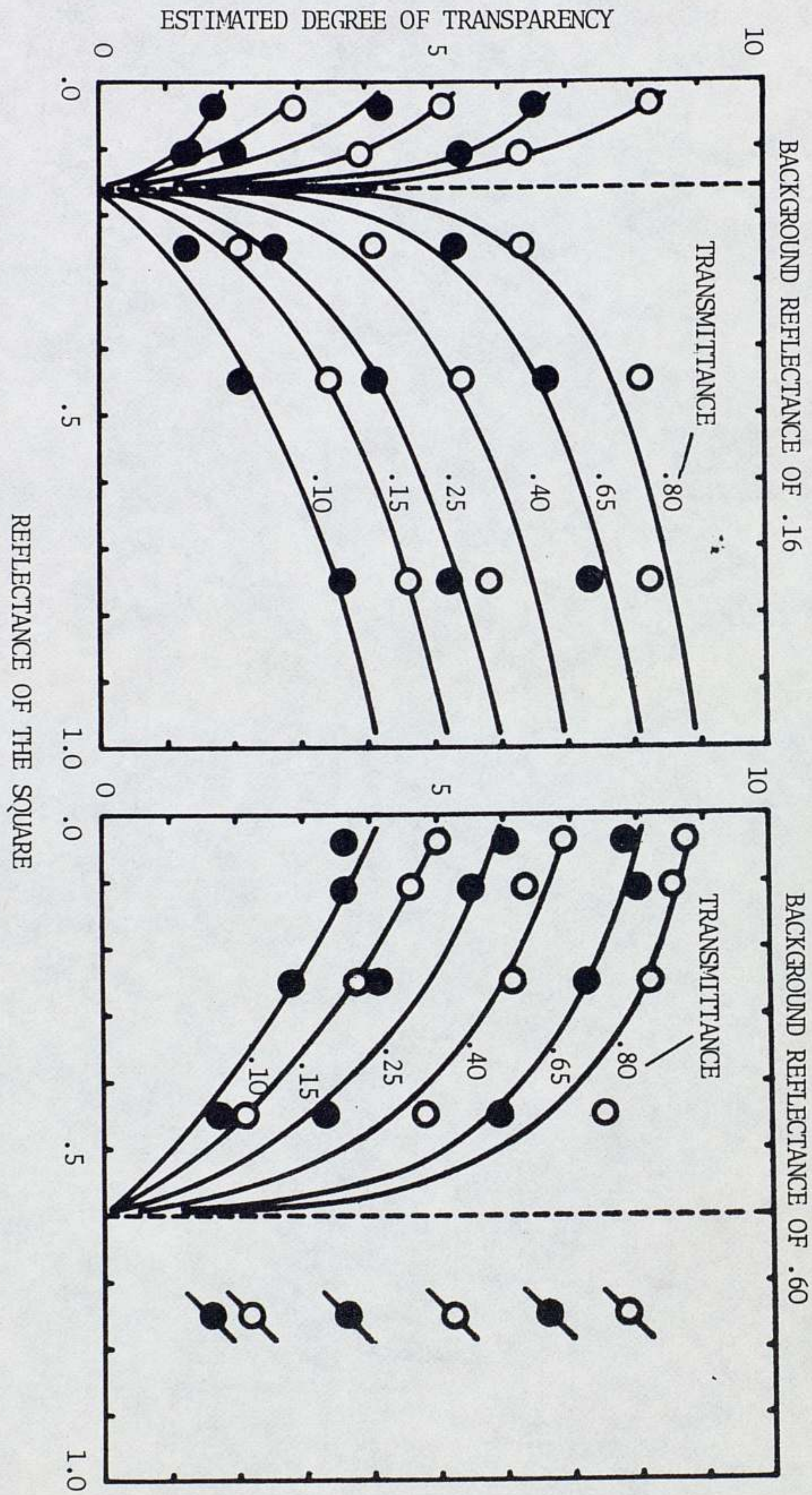


Figure 2

La trasparenza cresce con la trasmittanza e con la riflettanza dello sfondo

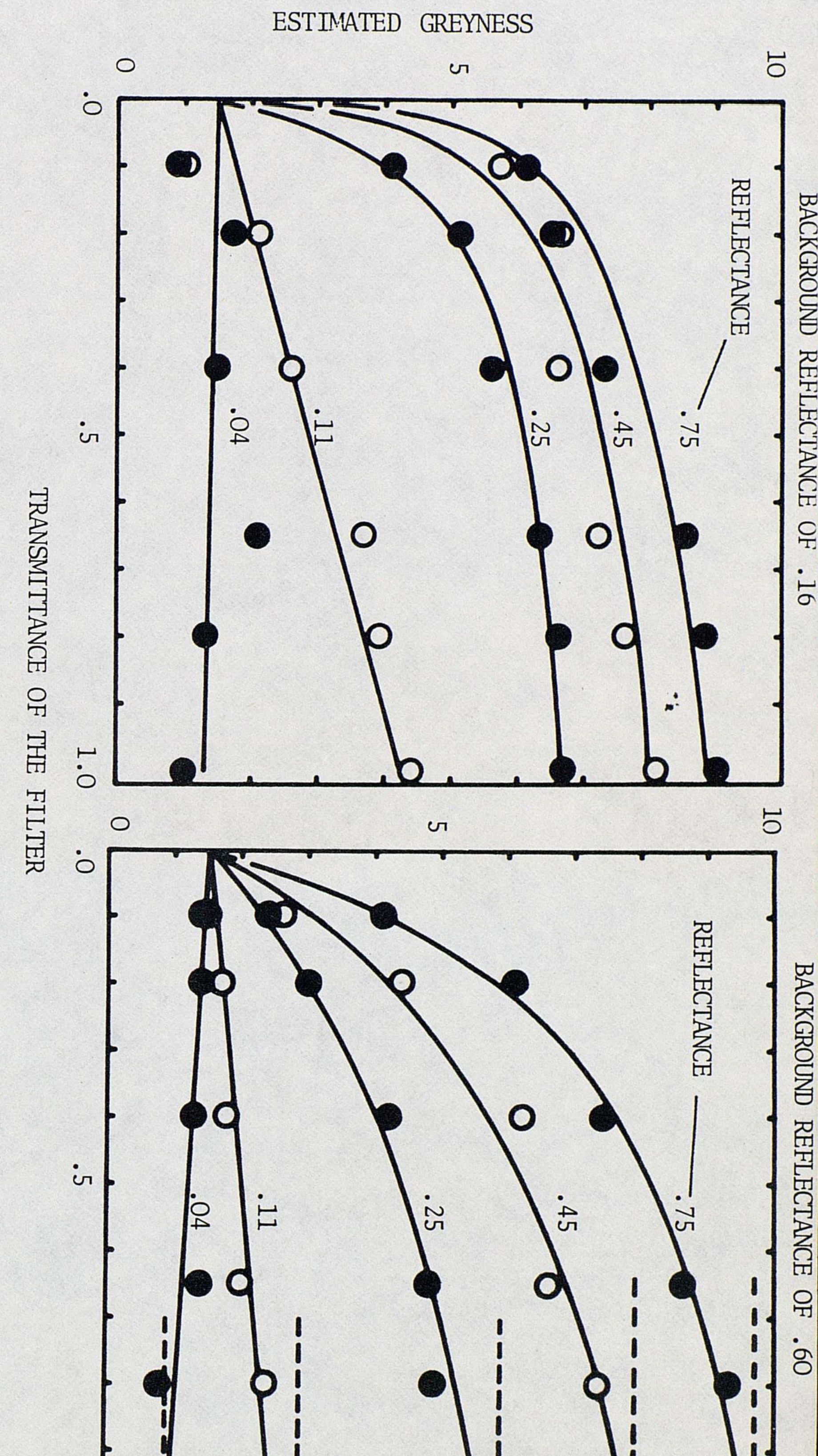
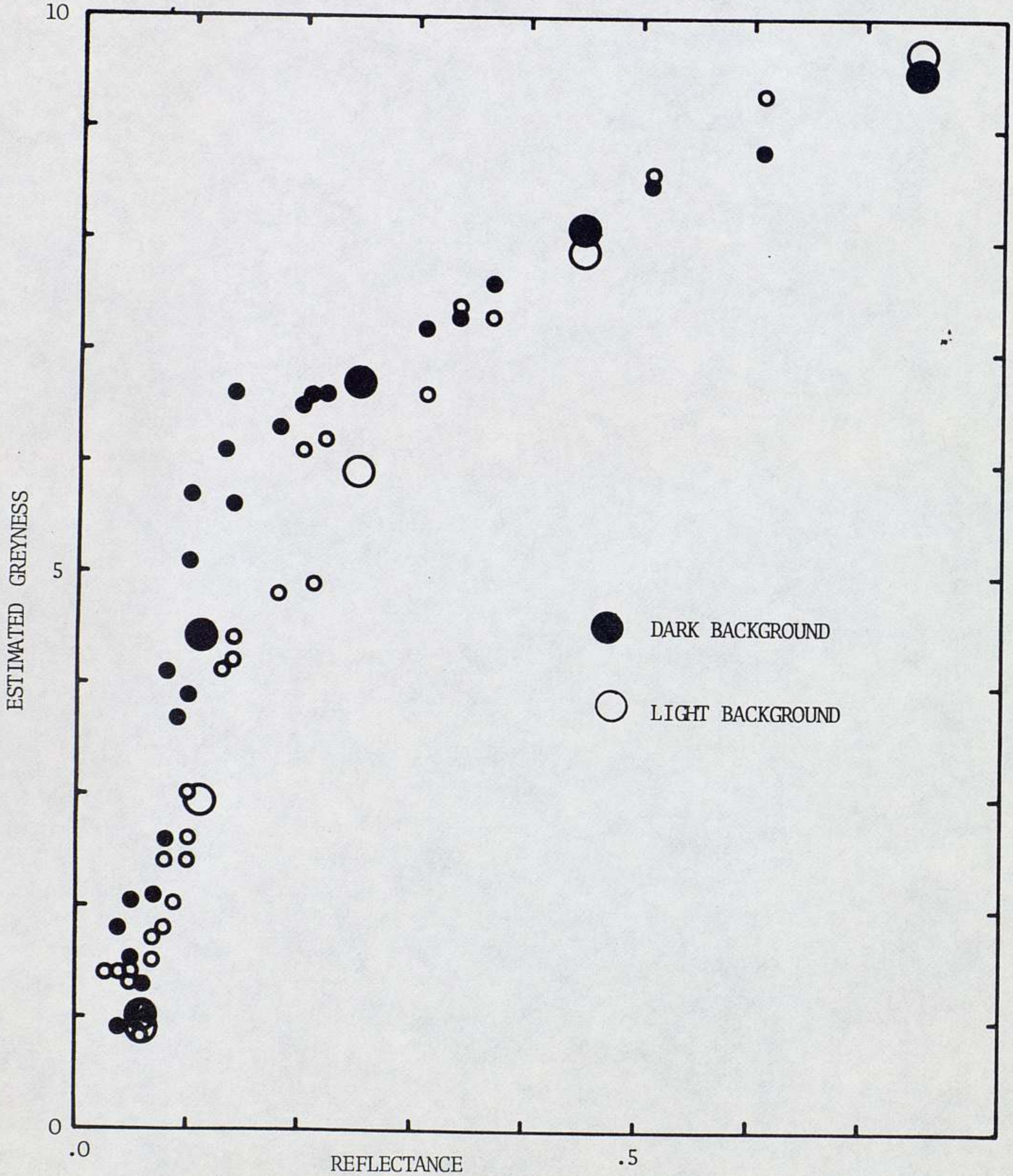


Figure 3

1 La chiarezza percepita cresce con la riflettanza della figura
2 " " " " con la trasmittanza del filtro
(solo per le figure chiare)
la differenza tra le chiarezze diminuisce col diminuire della ~~trasm~~ trasmittanza del filtro

Figure 4


10
5
0
.0
.5
ESTIMATED GREYNESS
REFLECTANCE
DARK BACKGROUND
LIGHT BACKGROUND

LEGENDS OF THE TABLES

Table 1

Mean estimates (20 subjects) of the degree of apparent transparency of the filter, for the background of reflectance .16.

Table 2

Mean estimates (20 subjects) of the degree of apparent transparency of the filter, for the background of reflectance .60.

Table 3

Mean estimates (10 subjects) of the grey of the square seen through the filter, for the background of reflectance .16.

Table 4

Mean estimates (10 subjects) of the grey of the square seen through the filter, for the background of reflectance .60.

LEGENDS OF THE FIGURES

Figure 1

Proximal stimuli produced by interposing a filter in between the observer's eyes and an object on a homogeneous background. Case 1 depicts the Fuchs phenomenon (transparency with three surfaces and no intersecting contours); case 2 depicts the most frequently studied kind of transparency (four surfaces with intersecting contours); and case 3 depicts the case in which opacity (Fuchs, 1923), or non-functional transparency (Masin and Idone, 1981), occurs.

Figure 2

The diagram on the left refers to the background of reflectance .16; the diagram on the right to the background of reflectance .60. The abscissae report the reflectance of the square on the background. The ordinates report the estimated degree of apparent transparency. Each curve corresponds to a different transmittance of the filter. The vertical broken line represents the reflectance of the background. As one can see, the estimated degree of transparency of the phenomenal filter decreases as the difference between the reflectance of the square and the reflectance of the background decreases.

Figure 3

The diagram on the left (right) refers to the background of reflectance .16 (.60). The abscissae report the transmittance of the filter. The ordinates report the estimated greyness of the square. Each curve corresponds to a different reflectance of the square. As one can see, the estimated grey of the various squares perceived beyond the filter, tends to a common value as the transmittance of the filter approaches zero. This common value is the colour of the filter (see section 5).

Figure 4

The big dots represent estimated points of a psychophysical function relating greyness to reflectance of the squares on the dark (filled dots) and light (unfilled dots) background. As to the meaning of the small dots, see the last two paragraphs of section 5.

AN EXPERIMENTAL COMPARISON OF THREE- VERSUS FOUR-SURFACE PHENOMENAL TRANSPARENCY

SERGIO CESARE MASIN
University of Padua, Padua, Italy

The three-surface transparency occurs when an object seen through a transparency does not jut out under the transparent surface. The four-surface transparency occurs when the object juts out. Observers rated the density of the transparent surface in both kinds of transparency. The results seems to show that the topological diversity between the two kinds of transparency has no functional significance. The stimulus conditions ruling the generation of the two phenomena were detected and expressed in terms of an algebraic model.

The basic proximal stimulus patterns producing phenomenal transparency are of two kinds. Figure 1 depicts these two possibilities when an object, the small square, is looked at through a filter, the big square. Figure 1a represents the case where the object does not jut out under the filter. Figure 1b represents the case of an object jutting out under the filter. In Figure 1a there are three proximal surfaces yielding the kind of transparency called here the *three-surface* phenomenal transparency. In Figure 1b there are four proximal surfaces yielding the *four-surface* phenomenal transparency.

==========
Fig. 1
==========

Theoretical interest has so far been addressed only to the four-surface transparency with achromatic colors. Metelli (1970, 1974a, 1974b, 1975; Masin, 1978) developed the following model. Phenomenal transparency entails the generation of overlapping surfaces. These surfaces are the transparent layer and the surfaces (the background and the objects on the background) seen through it. Moore-Heider (1933) hypothesized that, when transparency is perceived, the reduction colors[1] split, so to speak, into the colors of the corresponding overlapping surfaces. Let a, p, q, and b be the achromatic reduction colors corresponding to the proximal surfaces A, P, Q, and B in Figure 1b. On the basis of Moore-Heider's hypothesis, Metelli assumed that p splits into a and t, where t is the color of the transparent layer, and q splits into b and t, following a rule which is the opposite of Talbot's law. The model is stated as follows

$$p = \alpha_1 a + (1-\alpha_1)\, t_1, \qquad (1)$$
$$q = \alpha_2 b + (1-\alpha_2)\, t_2,$$

where the weights α_1 and α_2 are interpreted as coefficients expressing how transparent the transparent layer is. The weight α_1 (the symbol t_1) refers to the degree of perceived transparency (to the color) over the background, and α_2 (t_2) to the degree of transparency (to the color) over the part of the object under the transparent surface.[2] ?

We must ask now whether Model 1 applies also to the three-surface transparency (Figure 1a). Closely following Model 1 leads to the assertion that p splits into a and t, and q splits into x and t^1, where x is the color of the object seen through the transparency. Since x is an unknown, Model 1 is inapplicable, on principle, to the three-surface transparency. Moreover, Masin (1983) showed experimentally that x=q, which means that the process of splitting of reduction colors (Moore-Heider, 1933) does not occur in the three-surface transparency.

There exists, however, the possibility that the splitting of colors occurs in the four-surface transparency. It is clear that, in the case of this splitting occuring, it must be ascribed to the presence of the part of the object jutting out under the transparent surface (B in Figure 1b). It is, therefore, of theoretical importance to check whether this part has a *functional* influence on the degree of perceived transparency of the transparent layer.

EXPERIMENT 1

Method

Observers. The observers were 33 students and members at the Institute of Psychology.

Stimuli. Each of five (Kodak Wratten No. 96) achromatic gelatine filters, 72 x 76 mm, having a transmittance of .1, .2, .3, .5, or .8, was placed in between two exactly superimposed glass sheets of 1.5 x 220 x 275 mm. The reflectance of the filters was about .05. The two glass sheets, with the filter in the middle of them, were put in a wooden rectangular frame. A small achromatic square, 23 x 23 mm, having a reflectance .04, .11, .19, .26, .31, .40, .52, .67, or .87 (NCS papers, Sweden), was stuck at the center of each of nine achromatic backgrounds, 200 x 200 mm, all having the same reflectance .31.

The observer sat at a small table provided with a chin rest to keep the eyes level with the stimulus. On another table, the background with the small square on it was placed at 3.1 m from the observer's eyes. The wooden frames, with the filters suspended in the middle of them, were placed at 2.9 m, between the small square on the background and the observer's eyes. The observer viewed the scene binocularly. The background could be placed in two positions. In one position, the small square on the background was perceived in the middle of the filter (three-surface transparency, Figure 1a). In the other position, half of the square jutted out, on the left, under the filter (four-surface transparency, Figure 1b).

The wooden frame, parallel to the background, was tilted about 6 deg backwards so that the glass sheets reflected a uniform black curtain just above the observer's head, thus assuring no reflected image in the glass sheets. The light came from neon tubes on the ceiling. The illumination level was of 20 lux.

Procedure. The observers were shown each of the 45 combinations of filter and square on the background once, with the background in one position. Then the 45 combinations were again shown once with the background in the other position. All the 45 combinations were shown in random order, different for each observer. The experimenter was hidden by a screen. The patterns were shown by removing a screen. Two groups of 15 observers were formed. One group was first shown the three-surface transparency, and secondly the four-surface transparency. For the other group, this order was reversed.

For each combination, the observer was asked to assign a number to the filter according to its density, and write it down on a page of a notebook (one number per page). He was told to match the impression of quantity evoked by a number with the subjective quantity of density of the filter (that is, the greater the density the greater the number, the less the density the smaller the number), and could use any number that seemed the most appropriate. When observers asked, they were told they could use both integers and fractional numbers, of whatever magnitude they thought appropriate, and that they were not limited to using fixed upper and lower numbers.

Before starting the instructions, the observer was shown with seven photographs of a filter on a background made of four black and white squares. The transmittance of the original filter decreased in passing from photo 1 to photo 7. The experimenter described the figures saying that the density of the filter progressively increased in passing from photo 1 to photo 7, and that the greater the density the less the visibility of the background through the filter. All observers promptly agreed with this description. Afterwards they were shown two examples of a combination of filter and square in the experimental apparatus, one with a dense filter and one with a less dense filter (neither being the greatest nor the least densities). A session lasted 50 min.

Results

All observers used *spontaneously* a fixed range of numerical responses. For 29 observers the lower end number was 0, and the upper end number was 100 (N=12), 10 (5), 25 (2), 1200, 1000, 250, 200, 80, 50, 40, 30, 20, or 15. One observer used the range 30-40. The data were linearly transformed so as to have the common range 0-100 for all observers. This processing is admissable as all observers, except one, used the same upper and lower end numbers for both kinds of transparency. For the exceptional subject, the first number lower than the larger number was used as an upper number. ?

A 2x2x5x8 analysis of variance was carried out using the normalized numerical responses as scores. The factors were respectively, the group of observers, the kind of transparency, the transmittance, and the reflectance (the reflectance .31 was not included in the analysis).

The analysis showed no significant difference between the two groups of observers [$F(1,28)=1.17$]. Figure 2a shows the results pooled over the two groups ($N=30$). The main effects due to the kind of transparency are not statistically different [$F(1,28)=1.98$]. The interaction between the kind of transparency and the reflectance is significant [$F(7,196)=9.59$, $p<.001$].

==========
Fig. 2
==========

This interaction shows in Figure 2a as a departure from perfect overlapping of curves for the two kinds of transparency. This departure increases as the difference in lightness between the square and the background increases.

The group of 30 observers was formed by retaining in the group the observers who showed a pattern of results substantially similar to the one illustrated in Figure 2a. However, 3 observers were excluded from the group since their results, represented in Figure 2b, showed no systematic pattern.

Inspection of Figures 2a and 2b shows that observers rated the filters as having an apparent density (results represented by squares) of less than the highest possible density (100) even when the square on the background had a reflectance (.31) equal to the reflectance of the background. In this case, none of the observers detected any figure or inhomogeneity on the background. They were consequently expected to perceive the filter as opaque (Fuchs, 1923) and rate it as having the highest possible density, whatever its transmittance. However, only 19 out of 30 observers (Figure 2a) did perceive the filter as opaque. The remaining 11 observers plus the 3 observers excluded from the analysis (Figure 2b) detected the filter as transparent. A plausible reason for this is the following.

When a homogeneous filter is put on a homogeneous background, the filter appears opaque (Fuchs, 1923). Masin and Idone (1980) showed that, when the difference in lightness between the filter and the background is not too large, and when a favourable attitude is assumed, the filter may be perceived as transparent. The attitude towards perceiving transparency was induced by means of instructions to the observer. It is reasonable to suppose that, in the present experiment, the attitude was induced objectively by the experimental procedure. Observers felt a certain uneasiness in rating the density of a filter when no figure was perceived through it. The uneasiness was caused, they said, because the transparency they rated had something different in quality from that when a figure is detected through the filter. The existence of this form of transparency, which might be called *non-functional* transparency, has been independently reported by Tudor-Hart (1928, p. 283-284), Lauenstein (1943, p. 210), and Gibson (1975).

lavoro con filtri?

is very similar

The results represented by squares in Figure 2 are therefore misleading in that they reflect reponses both to opacity and to non-functional transparency. In order to check whether these kinds of responses were correlated with some other individual differences, the 19 observers who perceived opacity, and the 11 observers who perceived non-functional transparency, were assigned to two distinct groups, A and B. An analysis of variance, in which the new factor was group A vs. group B, was then carried out.

==========
Fig. 3
==========

The results are depicted in Figure 3. As may be seen, the curves for group B (N=11) are much flatter thah the curves for group A (N=19). The differences between the curves in the four graphs in Figure 3 are explained by significant interactions between kind of transparency and reflectance $[F(7,128)=7.44, p<.001]$, group A vs. group B and reflectance $[F(7,128)=18.82, p<.001]$, and group A vs. group B, transmittance, and reflectance $[F(28,728)=3.47, p<.001]$. The main effects due to the factor group A vs. group B are not statistically different $[F(1,26)=1.64]$.

Discussion

Tudor-Hart (1928) seems to have been the first who showed experimentally that apparent density of a filter, or an episcotister, increases as the sharpness of the contour of the object on the background decreases[3]. Her finding is confirmed by the results depicted in Figure 2a. dove?

The results in Figure 2a show that it makes a difference whether the case is one of 3- or 4-surface transparency. However, on assumption that apparent density depends upon the difference between p and q (Figure 1), the difference in results for the 3- and 4-surface transparency can be explained, reasonably, in terms of simultaneous contrast. In fact, owing to simultaneous contrast produced by B on Q, the difference p-q in Figure 1b increases (with respect to the difference p-q in Figure 1a) as the difference b-a increases. This could well explain the slight overestimation of apparent density in the case of the 4-surface transparency for large differences between a and b, and for low transmittances.

non c'è rapportato

hypothesis

This conclusion is of theoretical importance since it shows that the presence of the region B in Figure 1b has no functional effect. Only the difference p-q would be effective.

??

However, the absence of a pattern of results for three observers (Figure 2b) reveals the probable effect also of the difference, p-a, between the filter and the background. These results could be explained satisfactorily by assuming that the three observers rated the density only on the part of the filter over the background, while the other 30 observers rated the density of the filter over the figure. The three exceptional observers produced flat curves because, for a given filter, the difference p-a was nearly the same for all reflectances of the figure.

If there are 3 extreme cases where only transparency of the filter on the background is rated, then in the group of 30 observers there must be extreme cases where only transparency on the figure is rated, and reasonably also mixed cases. A plausible criterion to select the extreme and mixed cases from the group of 30 observers is the following.

The occurance of the impression of non-functional transparency depends only on the difference in color between the figure and the background (Masin and Idone, 1980). In the group of 30 observers, the observers whose response was based on this difference (i.e., p-a, when p=q and b=a), are likely to have rated non-functional transparency (mixed cases). It is also likely, although not necessarily, that the observers who did not rate non-functional transparency, were much more influenced by the difference p-q than by the difference p-a (extreme cases). Figure 3 represents the results for a subgroup (N=19) who did not report non-functional transparency, and for a subgroup (N=11) reporting non-functional transparency. From this analysis, it results that 19 out of 33 observers seemed to be prevailingly influenced by the difference p-q, 3 observers prevailingly by the difference p-a, and 11 observers by both differences.

gruppi formati a posteriori

AN AVERAGING MODEL OF RATED COLOR DENSITY

To sum up, the foregoing discussion leads to the following conclusion. In patterns like the ones in Figure 1, the variables by which the visual system produces the impression of transparency are 1) the difference in reduction color on the two sides of the contour separating the filter from the background (p-a), and 2) the difference in color on the two sides of the contour delimiting the object inside the filter (p-q). Objects or parts of objects jutting out under the filter do not enter functionally the perceptual transparency mechanism (they enter indirectly by producing concomitant color alterations, through simultaneous contrast and similar processes).

Let us now build a model to fit these results. It is first assumed that apparent density and difference in color are related by a power function. A power function is chosen because it describes a lot of sensory data (Marks, 1974). Let, therefore, the magnitude of apparent density of the filter evoked by the difference p-a be $\psi_{p,a}=k(p-a)^{\beta}$, and the magnitude evoked by the difference p-q be $\psi_{p,q}=k(p-q)^{\beta}$, where k and β are unknown parameters. The model proposed here states that the magnitude, ψ, of the overall impression of density of the filter is a weighted average of $\psi_{p,a}$ and $\psi_{p,q}$. A weighted average becomes necessary because observers spontaneously use a fixed range of numerical responses to rate apparent density, and ψ, $\psi_{p,a}$, and $\psi_{p,q}$ are consequently included in the same range. In symbols

$$\psi=k\left[w(p-a)^{\beta}+(1-w)(p-q)^{\beta}\right], \tag{2}$$

where w is an individual weight coefficient. For the 3 observers in Figure 2b, w=1. For the 19 observers in Figure 3, w fell close to 0. For the 11 observers in Figure 3, w fell somewhere in between 0 and 1.

The next two experiments had the purpose of looking for simplifications of Model 2. One important question is whether appropriate instructions are capable of rendering the individual exponent w equal to 0 or 1. If so, one term in Model 2 would vanish. ?

EXPERIMENT 2

Method

Observers. The observers were 10 students and members at the Institute of Psychology, different from those used in Experiment 1.

Stimuli. The stimulus conditions were the same as in Experiment 1, except for the following changes. Three filters with a transmittance of .1, .3, or .5 were used. Sixteen backgrounds with a square in the middle were built. There were 4 sets of 4 backgrounds with the same reflectance. In each set the reflectance of the square was varied as follows.

Reflectance of the background	.04	.19	.35	.76
	.06	.23	.31	.67
Reflectance of	.13	.31	.26	.52
the square	.35	.52	.19	.31
	.76	.87	.08	.13

Each background was placed always in the same position, so that the square on it was perceived in the middle of the filter (3-surface transparency).

Procedure. The procedure was the same as in Experiment 1, except for the following changes. Five observers were shown once each of the 48 combinations of the filter and the square (figure) on the background, and were asked to rate the apparent density of the filter over the figure. Then the 48 combinations were again shown once, and the observer was asked to rate the apparent density of the filter over the background, disregarding the figure. For the other five observers, this order was reversed. The rating had to be performed using the numbers in the range 0-100. The number 0 represented the case of perfect transparency, as for example in the case of a colorless pane. The number 100 represented the case of opacity. Before the rating began, the observer was given no information as to the range of apparent densities. As a single example of an experimental pattern, all observers were shown the same combination of filter (transmittance .3), square (reflectance .52), and background (reflectance .76).

Results and discussion

The results are depicted in Figure 4. The upper four graphs refer to the estimate of apparent density over the background (unbroken lines). The lower graphs refer to the estimate of apparent density over the figure (dashed lines). No statistical test was applied since inspection of individual results showed that all observers produced the same pattern of results.

The curves in the first upper graph superimpose around a density of 50. These curves represent the means for 9 observers. One observer was excluded from computations, in this particular case, since all his numerical responses were practically 0. Another observer declared that she could assign to a single filter 0 or 100 just as well. In point of fact, when a black filter is superimposed on a black background, a very ambiguous impression ensues. This, therefore, is a special case that should not be taken into account here.

==========
Fig. 4
==========

As may be seen in Figure 4, the curves in the upper graphs are flatter than the curves in the lower graphs. However, the upper curves are not as flat as the curves for the 3 observers in Figure 2b, whose w=1. Therefore, it may be concluded that the present instructions do have an effect on w. However, it seems that w cannot be made exactly equal to 1, and consequently Model 2 cannot be simplified in the way hoped for.

istruzioni = compito ?

EXPERIMENT 3

If it is assumed that $\beta=1$ in Model 2, and if p and a are kept constant, ψ must vary linearly with q. In the present experiment, observers were asked to rate both ψ and q, in order to check whether or not the relation between these two phenomenal variables is linear, and consequently $\beta=1$, for p and a constant.

Method

Observers. The observers were 20 students and members at the Institute of Psychology, different from those used in Experiments 1 and 2.

Stimuli. This experiment was carried out in a different room. The stimulus conditions were the same as before, except for the following changes. The eye distance from the filter was 1.3 m. The illumination level was of 15 lux. Three filters with a transmittance of .1, .3, or .8 were used. Eight backgrounds, all having the same reflectance .23, were built. A square (figure) was stuck in the middle of each background. The reflectance of a square was .26, .31, .35, .40, .46, .59, .67, or .87. Filters and figures were combined to produce 24 cases of 3-surface transparency.

Procedure. The procedure was the same as before, except for the following changes. Observers were first shown once the 24 combinations of filter and figure, and asked to rate apparent density. Then they were shown again once the 24 combinations, in a different random order, and asked to rate the whiteness of the figure as seen through the filter. Masin (1983) found that, under these instructions, observers spontaneously estimate the reduction whiteness. The other ten observers were first asked to rate whiteness, and then apparent density.

Observers had to use the numbers from 0 to 100. In the case of apparent density, 0 represented perfect transparency, and 100 opacity. Numbers in between 0 and 100 had to be assigned to each filter proportionally. That is, the greater the density the larger the number. In the case of whiteness, 0 represented the blackest black, and 100 the whitest white, ever experienced in observer's life. Numbers in between had to be assigned to the whiteness of figures proportionally. That is, numbers lower than 50 to dark grays, numbers larger than 50 to light grays, and the number 50 to the figure having a gray color exactly between black and white. As a single example of a stimulus pattern, all observers were shown the same combination of filter (transmittance .3), and square (reflectance .46) on the background. The observer was given no information as to the stimulus ranges. Each session lasted about 30 min.

Results and discussion

The results for each filter are depicted in Figure 5. The ratings of color density are represented on the ordinate, and the ratings of whiteness are represented on the abscissa. Using the least squares method, a straight line was fitted through the data points. The size

========
Fig. 5
========

of standard errors, represented by the vertical and horizontal bars, indicates that a straight line is, at least, a very good approximation of the true fitting curve. This strengthens the assumption that $\beta=1$, and allows a simpler statement of the model, which now is

$$\psi=k[w(p-a)+(1-w)(p-q)]. \qquad (3)$$

Note that the slope of the straight line for the transmittance .8 is consistently different from the slope for the transmittance .1 $[t(19)=3.65,\ p<.002]$. This shows that, in this experiment, the weight w in Model 3, besides varying between observers, also varied within a single observer as a function of p and q.

CONCLUSION

So far, no straightforward experimental demonstration of the tenability of Model 1, at a level of measurement higher than the ordinal, has been given. As stated in the introduction to Experiment 1, Model 1 is inapplicable to the 3-surface transparency. The results obtained in Experiment 1 (Figure 2a) show that the rated density of the transparent surface is functionally independent of the topologocal difference between the 3- and 4-surface transparency. This would imply that Model 1 is not even applicable to the 4-surface transparency.

To summarize, the empirically-based Model 3 applies both to the 3- and 4-surface transparency, while Model 1 applies, at most, only to the 4-surface transparency. Both Models 1 and 3 are averaging models. However, Model 3 is more parsimonious in that it contains only w as an unknown parameter (k should be considered as a constant dependent on the unit of measurement), while Model 1 contains four unknown parameters. The results obtained in Experiments 1 and 2 show that the apparent density of the transparent surface depends on the observer's attitude. The weight w in Model 3 encodes the observer's preference for the difference in reduction whiteness between the transparent surface and the figure, with respect to the difference in reduction whiteness between the transparent surface and the background, while Model 1 does not take into account any individual differences.

REFERENCES

FUCHS, W. Experimentelle Untersuchungen über das simultane Hintereinandersehen auf derselben Sehrichtung. *Zeitschrift für Psychologie*, 1923, 91, 145-235.

GIBSON, J.J. Three kinds of distance that can be seen, or how Bishop Berkeley went wrong. In G.B.F. D'Arcais (Ed.), *Studies in perception*. Milano: Martello, 1975, p. 83-87.

GYULAI, E. Grado, colore e uniformità della transparenza fenomenica. *Atti dell'Istituto Veneto di Scienze, Lettere ed Arti*, 1976, 134, 791-803.

KANIZSA, G. *Organization in vision*. New York: Praeger, 1979.

KATZ, D. *Die Erscheinungsweisen der Farben und ihre Beeinflussung durch die individuelle Erfahrung*. Leipzig: Barth, 1911.

LAUENSTEIN, O. Schattencharakter und Dingcharakter. *Zeitschrift für Psycgologie*, 1943, 154, 203-214.

MARKS, L.W. *Sensory processes*. New York: Academic Press, 1974.

MASIN, S.C. A contribution to the theory of perceptual scission in phenomenal transparency. *Italian Journal of Psychology*, 1978, 5, 171-191.

MASIN, S.C. A psychophysical study of the Fuchs phenomenon. *Perception*, 1983, in press.

MASIN, S.C., & IDONE, A.M. Studio sperimentale sulla percezione della trasparenza con figura e sfondo acromatici e omogenei. *Giornale Italiano di Psicologia*, 1980, 8, 265-277.

METELLI, F. An algebraic development of the theory of perceptual transparency. *Ergonomics*, 1970, 13, 59-66.

METELLI, F. The perception of transparency. *Scientific American*, 1974 a, 230 (4), 90-98.

METELLI, F. Achromatic color conditions in the perception of transparency. In R.B. Macleod and H.L. Pick (Eds.), *Perception*. Ithaca, N.Y.: Cornel University Press, 1974 b, p. 95-116.

METELLI, F. On the visual perception of transparency. In G.B.F. D'Arcais (Ed.), *Studies in perception*. Milano: Martello, p. 445-487.

MOORE-HEIDER, G. New studies in transparency, form, and colour. *Psychologische Forschung*, 1933, 17, 13-55.

TUDOR-HART, B. Studies in transparency, form, and color. *Psychologische Forschung*, 1928, 10, 255-298.

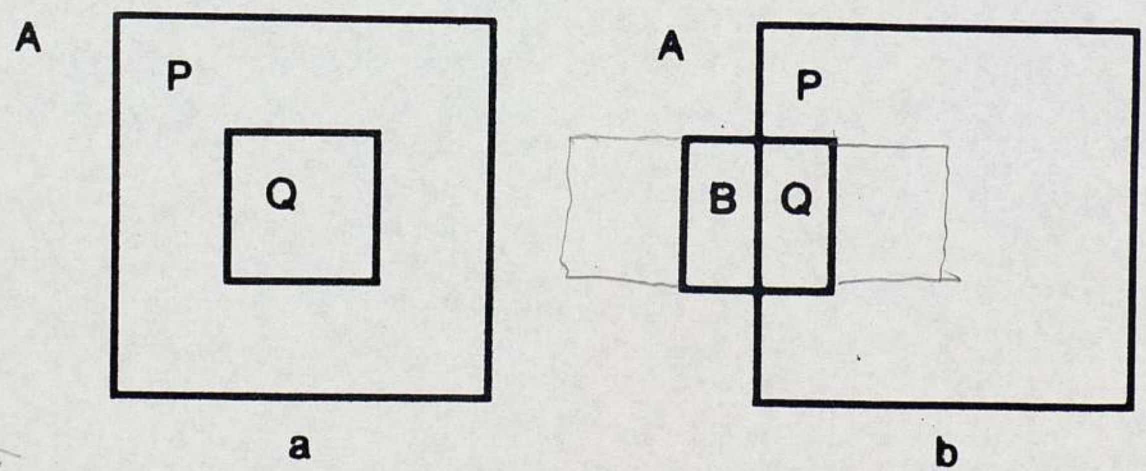
NOTES

[1] The reduction color corresponding to a given region of a transparent pattern (i.e., Q in Figures 1a or 1b) is the color 1) when the region is looked at while assuming an extreme analytical attitude (Fuchs, 1923); 2) when transparency is abolished by altering the so-called topological or figural conditions of transparency (Kanizsa, 1979); or 3) when the region is looked at through the hole of a reduction screen producing, in the hole, the same amount of simultaneous contrast as that produced, in the region looked at, when there is no reduction screen.

[2] In the special case where the degree of transparency and the color of the transparent surface are phenomenally homogeneous over the entire surface, $\alpha=\alpha_1=\alpha_2$ and $t=t_1=t_2$, and the system of equations constituting Model 1 can be solved for both α and t (Metelli, 1970).

[3] Katz (1911) proposed a similar explanation for the perception of mist. According to Katz, when one looks through a filter covering the entire visual field, an impression of mist ensues. The thickness of the mist increases as the transmittance of the filter decreases. The condition for the diminution of the thickness of the mist would be the reduction in clearness, or sharpness, of the surface contours.

The results of an experiment by Gyulai (1976) also stress the importance of the difference in reflectance between adjacent proximal surfaces. In her experiment, patterns made of gray paper were used. The results can be rephrased as follows. If episcotisters with the same sector size have colors varying from black to white, and the same bicolored background is looked at through each episcotister revolving at fusion speed, the observers, who are asked to arrange the episcotisters in order of increasing apparent density, produce two opposite dispositions. Half the observers order the episcotisters from black to white, and half the observers from white to black. The occurance of a given disposition depends upon which part of the background the observer chooses to look at through the episcotister. The less the difference in (reduction) color between the background and the episcotister, the less the judged density. Gyulai did not extend the experiment to episcotisters having different sector sizes.



verità
del tutto P
risultà senza specificare
dove

FIG. 1

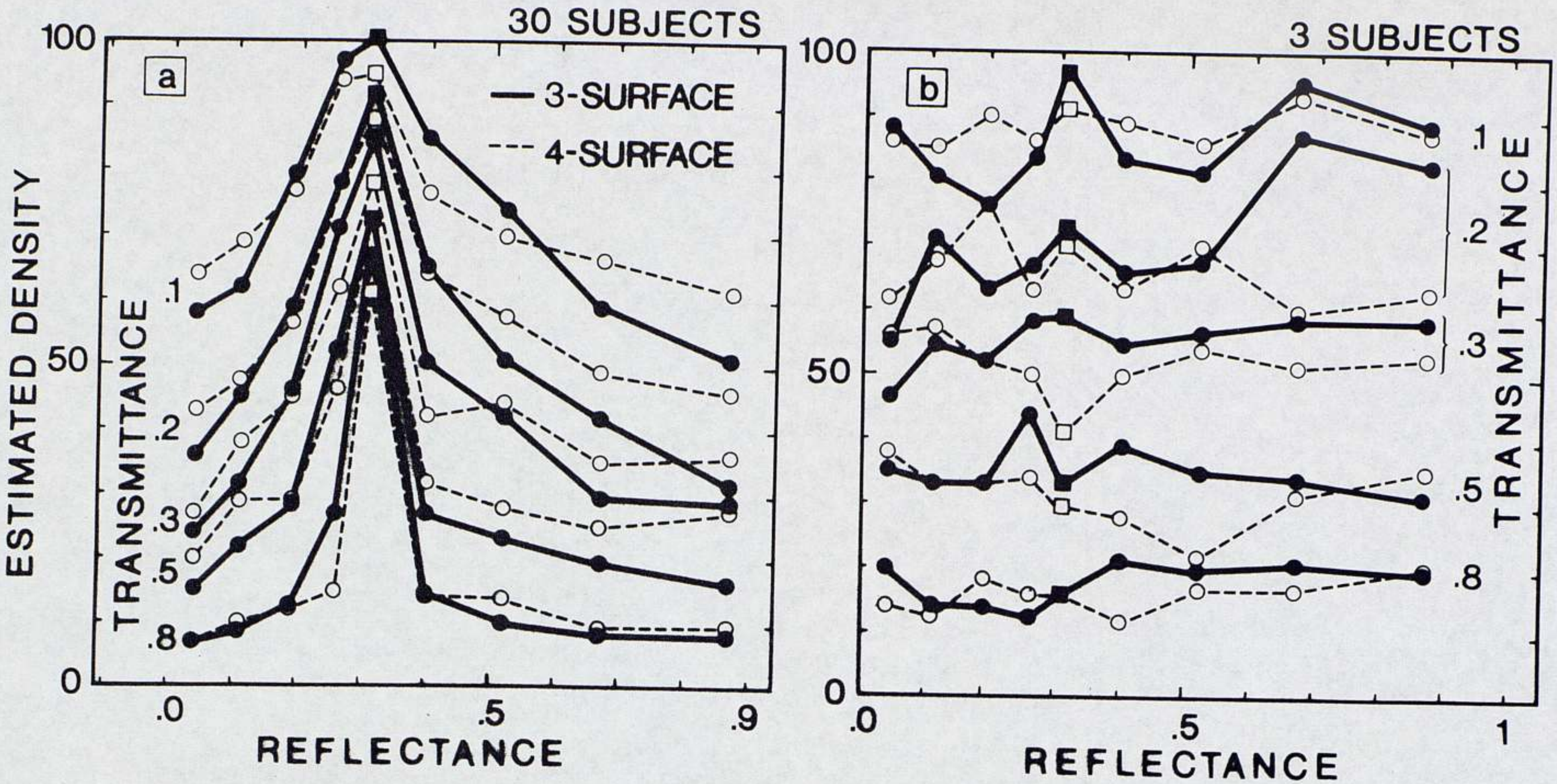

30 SUBJECTS
a
3-SURFACE
4-SURFACE
ESTIMATED DENSITY
TRANSMITTANCE
100
50
0
.1
.2
.3
.5
.8
.0
.5
.9
REFLECTANCE
3 SUBJECTS
b
100
50
0
.1
.2
.3
.5
.8
TRANSMITTANCE
.0
.5
1
REFLECTANCE

FIG. 2

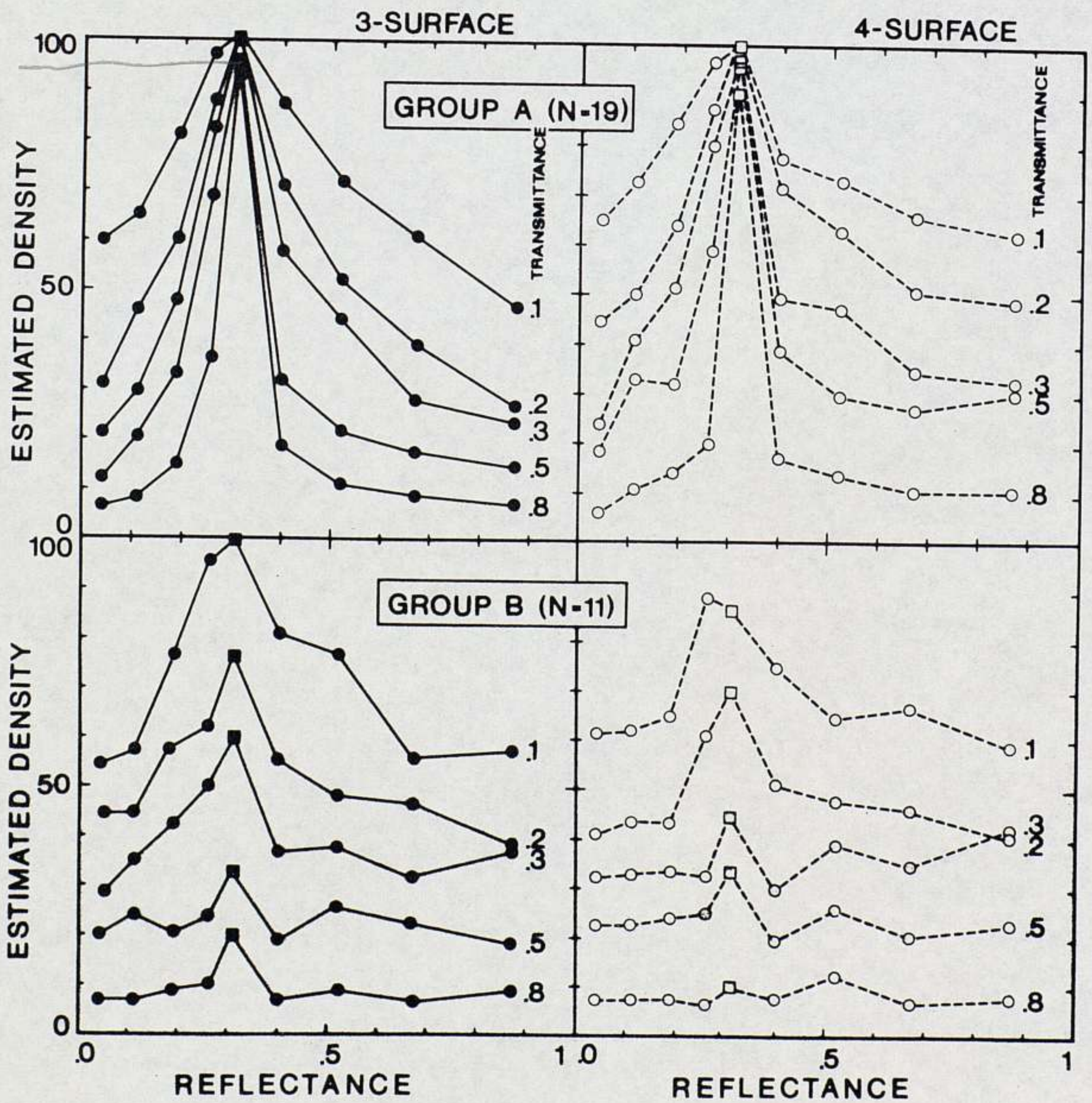

3-SURFACE
4-SURFACE
GROUP A (N-19)
GROUP B (N-11)
ESTIMATED DENSITY
TRANSMITTANCE
REFLECTANCE
100
50
0
.0
.5
1
.1
.2
.3
.5
.8

Fig. 3

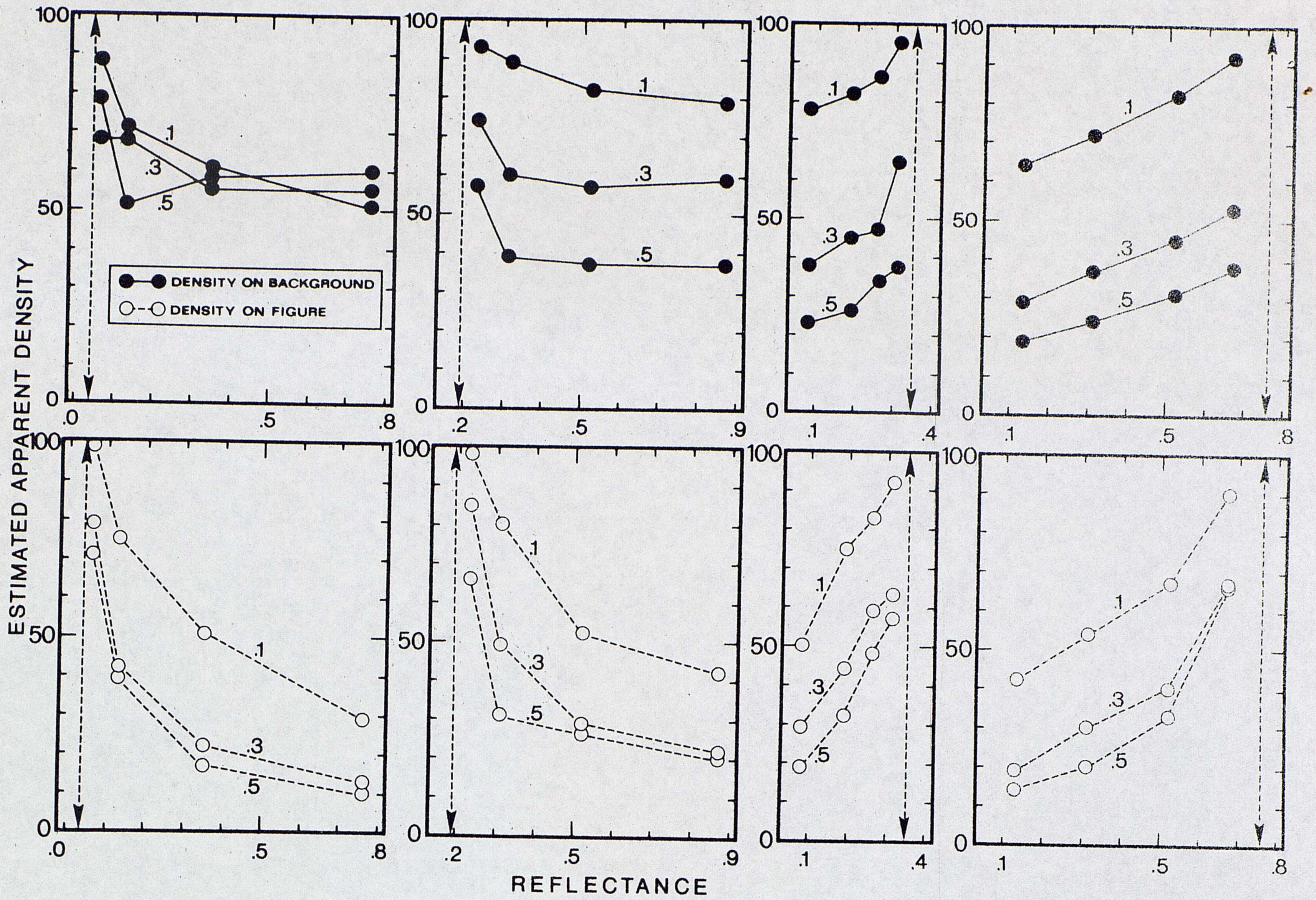

ESTIMATED APPARENT DENSITY
DENSITY ON BACKGROUND
DENSITY ON FIGURE
.1
.3
.5
100
50
0
.0
.5
.8
.2
.9
.1
.4
REFLECTANCE

Fig. 4

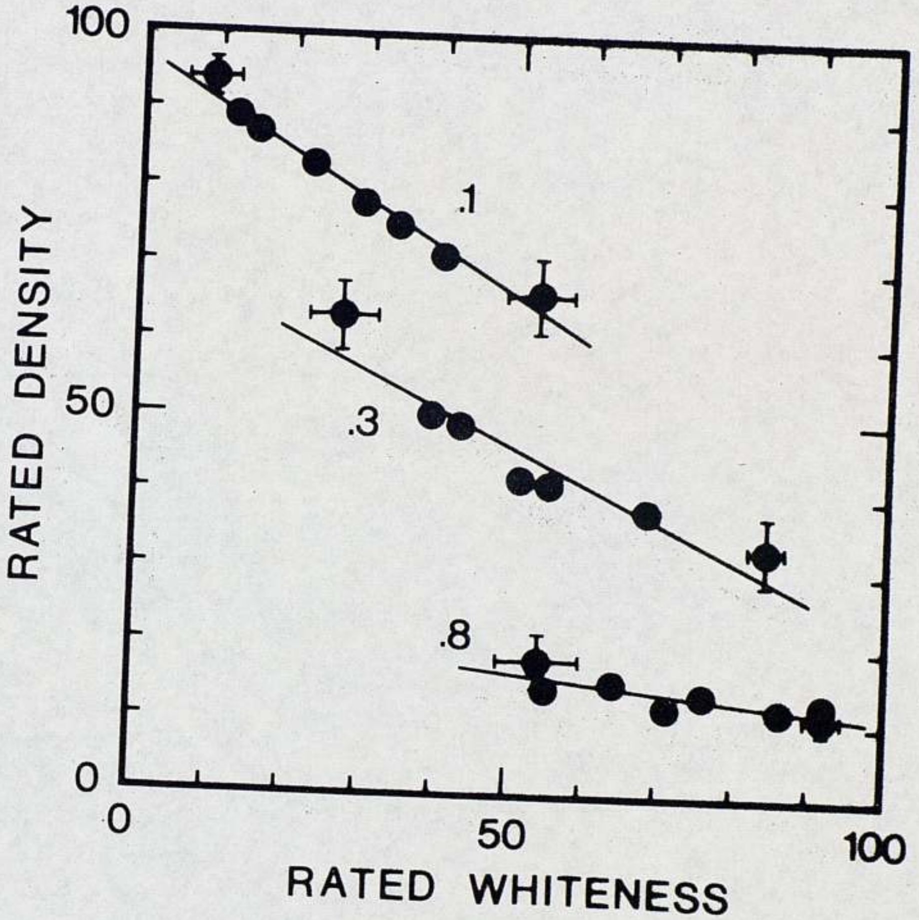


Fig. 5

CAPTIONS

Figure 1

Illustration of the three- (a) and four-surface (b) transparency. The three-surface transparency occurs when the object (the small square) does not jut out under the transparenct surface (the big square). The four-surface transparency occurs when the object juts out.

Figure 2

The results in Figure 2a show that apparent density of a transparent surface is a function of the difference in reflectance (and consequently in color) between the figure and the background. Figure 2b shows the results for 3 special observers. Observers rated the density both in the case of the three- (unbroken lines) and four-surface (dashed lines) transparency.

Figure 3

The same results as in Figure 2a when observers are subdivided in two subgroups according to whether or not they detected non-functional transparency.

Figure 4

Estimated apparent density as a function of reflectance of square on background, for a transmittance of .1, .3, or .5. The reflectance of the background is indicated by the arrows. The upper (lower) graphs represents the results when the observer estimated the density on the background (figure).

Figure 5

Relation between apparent density of filter and whiteness of figure when, for a given filter with transmittance .1, .3, or .8, the difference in color between filter and background is kept constant.

Caro Masin,

vedo con piacere che ti sei reso conto che, così com'era, la tua for= mula non aveva senso , e che le differenze andavano prese in senso assoluto. Ma ci sono ancora molti punti che offrono il fianco alle critiche, sia relativamente alla formula , sia relativamente ad al= tri aspetti dei tuoi due ultimilavori. Qui mi limito ad occuparmi della formula

I. <u>La formula per la misura della densità del filtro</u>

1.Masin parte da tre presupposti

a) che l'opacità, o inversamente la trasparenza, dipenda dal= la differenza di chiarezza fra la regione trasparente (nel caso dell' effetto Fuchs, il filtro), e la regione vista per trasparenza, e cioè

$$\Psi = f\ /p-q/,\ /p-a/$$

b) che la formula debba essere una funzione di potenza, "per= chè le funzioni di potenza descrivono una quantità di dati sensoria= li". L'argomentazione è molto povera, e corrisponde a un "si dice"; comunque Masin dimostra elegantemente che in questo caso la potenza è 1, cioè,in parole povere, non si tratta di una funzione di potenza.

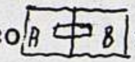
c) che l'impressione del grado di densità del filtro è data dalla media ponderata delle due funzioni f /p-q/ ed f /p-a/.

A questo proposito sorge naturale l'obiezione che se vi è opa= cità completa su uno dei due versanti, il calcolo della media ponde= rata non appare sensato.

Ma è strano che Masin non si sia reso conto che, ammesso che le due funzioni misurino quello che hanno il compito di misurare, ha a disposizione un prezioso strumento per misurare la densità o la trasparenza rispetto a due diverse regioni viste per trasparenza, cioè non soltanto le regioni A e Q del caso Fuchs, ma in altri casi, in cui tale misura presenta particolare interesse.

2. La formula proposta non sembra misurare la densità del filtro.

Se p=q l'opacità è completa, e quindi la misura Ψ della densità deve aumentare. Invece, annullandosi uno dei termini della media ponderata, la misura della densità diminuisce. Analogamente, se la differenza fra <u>p</u> e <u>q</u> diminuisce, aumenta l'opacità del filtro, ed invece la misura Ψ dell'opacità diminuisce.

3. La formula proposta, ove funzioni, può servire soltanto per misurare la densità del filtro (con le limitazioni indicate in l.c) nell'effetto Fuchs. Non è una formula generale. Altrimenti, nel caso paradigmatico  non si capisce perchè si dovrebbe tener conto di A e non di B.

4. Non si vede perchè f/p-a/ ed f/p-q/ debbano ridursi semplicemente a w /p-a/ + (1-w) /p-q/. Potrebbe trattarsi di una qualsiasi altra espressinne, anche se non di potenza.

5. L'equazione ha due incognite, ψ e W. Occorre quindi una seconda equazione.

5. Ci sono altre proprietà utili che un'equazione di questo genere avrebbe, e che non sono state messe in luce, e cioè

a) la possibilità di controllare se la trasparenza è equilibrata

b) la possibilità di misurare la trasparenza nei casi di trasparenza non equilibrata.

6. Il fatto che la differenza tra la trasparenza nell' effetto Fuchs e la trasparenza nella situazione di Fuchs modificata facendo sporgere la superficie Q dal filtro sia trascurabile, anzichè dimostrare (non si sa perchè) l'inapplicabilità del modello di Metelli anche nelle situazioni a 4 superfici, dimostra come il modello di Metelli sia applicabile anche all'effettoFuchs.

A Masin

1. Il colore della figura vista il filtro risulta pari al colore di riduzione.
L'atteggiamento assunto dal soggetto al quale si chiede di valutare il colore di un quadrato relativamente piccolo al centro del campo è tale da isolare il quadrato dal resto fornendo un giudizio sul colore di riduzione

2. L'effetto Fuchs, che per realizzare la trasparenza richiede la bidimensionalità, rappresenta un caso anomalo. In effetti dai risultati ottenuti facendo valutare la trasparenza nelle due situazioni a 3 e a 4 superfici non si può ricavare nessuna conseguenza perchè nei due casi il grado di trasparenza risulta diverso, anche se tale diversità può essere imputata (ma solo in parte) ad un effetto di contrasto. Ma non è legittimo considerare il caso anomalo come prototipo e generalizzare a partire da questo. Più legittimo, più naturale sarebbe tentare di spiegare il caso anomalo per mezzo del caso normale.

3. La formula costruita da Masin al posto di quella in uso presta il fianco alle seguenti obiezioni

$$\psi = K[W(p-a) + (1-W)(p-q)]$$ dovrebbe essere una misura della densità dello strato trasparente

a) Si sa che se $p=q$ non c'è trasparenza; in questo caso ψ dovrebbe essere massimo (opacità) invece si ha $K[W(p-a)]$ cioè un valore inferiore dei casi di trasparenza.

b) se la differenza fra p e q diminuisce, fermo restando il resto, aumenta l'opacità; mentre invece ψ diminuisce

c) La formula non ha né un massimo né un minimo. Il minimo non è zero, perché φ può assumere valori negativi. E che significato hanno questi valori negativi?

d) La formula si può applicare eventualmente alla situazione a 3 superfici perché manca il termine b. E' c'è un termine che variando può far passare dalla trasparenza all'opacità

4. Nel caso della polemica Koffka-Bourdon ci troviamo a dover scegliere fra un ricercatore geniale che presenta una spiegazione elegante e un pedante che cerca di banalizzare il fenomeno con una spiegazione abituale. Entrando nei particolari vedremo che Bourdon sostiene a) un effetto di contrasto in un diverso piano, ciò che contrasterebbe a quanto osservato da Wolff, Beary ecc. b) che avvicinando al massimo lo sfondo all'episcotista si abolisce la trasparenza, cosa che non corrisponde in quanto si ha trasparenza anche con mosaici, casi in cui non c'è nessuna distanza× fra velo trasparente e sfondo. Il contrasto sembra essere la chiave che apre tutte le serrature, come l'esperienza passata. Comunque bisogna almeno concludere, se proprio non si vuole accettare lo sdoppiamento fenomenico, che occorre ancora sperimentare con i colori[1], prima di decidere.

[1] p. es. blu e rosso anziché blu e giallo

× o solo una distanza virtuale

d) Variando, in Fig. 2b, ~~la~~ il colore della parte protrudente B si può passare dalla trasparenza all'opacità. Ma siccome b non è una delle variabili della formula di Metelli, questo fatto non influisce sull'indice calcolato ~~in questo modo~~ con una formula che esclude b.

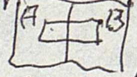
Il fatto che la differenza fra la trasparenza nell'effetto Fuchs e la trasparenza nella situazione di Fuchs modificata ~~si~~ sia "trascurabile" ~~dimostra~~ anziché dimostrare (non si sa perché) l'inapplicabilità del ~~mio~~ modello di Metelli anche nelle situazioni a 4 superfici, dimostra che il modello di Metelli è applicabile anche all'effetto Fuchs.
~~Basta infatti far sporgere~~

3. C'è una serie di proprietà utili che un'equazione di questo genere avrebbe e che non sono state messe in luce
a) la possibilità di controllare se la trasparenza è equilibrata
b) la possibilità di misurare la trasparenza nei casi di trasparenza non equilibrata

4. Si dovrebbe ~~controllare~~ per ~~scoprire~~ appuntamenti ~~esattamente~~ trovare la funzione che dà una misura esatta.

5. L'equazione così com'è vale soltanto per l'effetto Fuchs. ~~Un'equaz~~ Usarla per le altre situazioni di trasparenza non sarebbe giustificato. Perché non tener conto di B? E allora quali sarebbero i termini tra i quali porre la media ponderata?

~~5~~ ~~occorre un'equazione per calcolare W, perché~~ l'equazione ha due incognite φ e W. Quindi è necessaria una seconda equazione.

3. La formula proposta, ove funzioni, ~~serve~~ più soltanto con la limitazione indicata in 2 e per misurare la scurità del filtro nell'effetto Fuchs. Non è una formula generale, che dovrebbe tener conto anche della regione B. Altrimenti nel caso paradigmatico  non si capisce perché dovrebbe tener conto di A e non di B.

4. Non si vede perché $f(p-q)$ e $f(p-a)$ devono ridursi semplicemente a $w f(p-q) + (1-w)(p-q)$. Potrebbe trattarsi di una qualsiasi funzione di $(p-q)$ e $(p-a)$; la funzione utile potrà se mai venir trovata per tentativi successivi.

Perché non p. es. $\frac{1}{p-q}$

$|a-p|$ e $|b-q|$

3. L'equazione è la media ponderata di due trasparenze (opacità). Ma che senso ha questa media? Se $p=q$ non c'è trasparenza nella parte interna ma soltanto nella parte esterna. L'equazione prevede una opacità media, ~~mentre~~ risultati che non rispondono alla realtà. Se mai vanno tenute separate le due equazioni $w|a-p|$ e $(1-w)|b-q|$.

~~Da notare che $|p-q|$ dà zero, quindi l'equazione non è opacità~~

4. Come si calcola w? Le incognite sono z, ψ e w. L'equazione di Metelli dà soltanto il valore di ψ. Occorre un'altra equazione per w.

5 L'asserzione che la formula di Metelli ha 4 incognite mentre quella di Masin ne ha una sola è completamente falsa. Nella maggior parte dei casi (sempre, coll'opacità) il sistema di due equazioni ha soltanto due incognite ed è risolvibile. Altrimenti il sistema di 2 equazioni non sarebbe stato mai postulato. Quanto alle equazioni di Masin, ha due incognite e finché non ~~se~~ troveremo una seconda equazione, è insolubile.

L'indice di opacità
di S. C. Masin

In origine Masin aveva proposto e difeso a spada tratta la formula $\varphi = K[W(p-a)+(1-W)(p-q)]$ e poi finalmente si è accorto che bisognava prendere $(p-a)$ e $(p-q)$ in senso assoluto, cioè $|p-a|$ e $|p-q|$ per evitare che la formula potesse assumere valori negativi, senza che nel ~~?~~ passaggio dal valore positivo al valore negativo si potesse attribuire un particolare significato.

Anche nella sua nuova forma, l'equazione ha carattere arbitrario. Perché una media ponderata e non, p.es., un rapporto?

Il rapporto sarebbe $\frac{p-a}{p-q}$ oppure $\frac{a-q}{a-p}$ che è l'equazione di α per la trasparenza parziale. O una qualsiasi funzione di a, p, q?

2 Ma anche dopo la rappezzatura la formula non regge.

È chiaro che la formula, così com'è può tutt'al più servire al caso Fuchs, cioè con 3 superfici interessate al fenomeno. Estendendolo ci sarebbe la scelta fra $|p-q|, |a-b|, |a-p|, |a-q|, |b-p|, |b-q|$.

Volendo applicare il criterio usato per l'effetto Fuchs, si dovrebbero considerare le superfici che sono in rapporto di trasparenza, cioè

Osservazioni sulla formula proposta da Marin per misurare la trasparenza

1. Marin parte da tre presupposti

a) che la trasparenza dipenda dalla differenza di chiarezza fra la regione "trasparente" e la regione "vista per trasparenza" (~~presupposto fondato in base ai risultati ottenuti con la formula $\frac{p-q}{a-b}$ e cioè fermo restando a-b, la trasp~~ (presupposto fondato su dati sperimentali, e cioè

$$\psi = f|p-q|, |p-a|$$

b) che le formule psicofisiche sono "di solito" ~~equazioni~~ funzioni di potenza. (L'argomentazione è molto povera e corrisponde a un "si dice". Comunque Marin dimostra elegantemente che in questo caso ~~non si tratta di una funzione di potenza~~ la potenza è 1, cioè, in parole povere, non si tratta di un'equazione di potenza.

c) che l'impressione della densità del filtro è data dalla media ponderata delle due funzioni $f|p-q|$ e $f|p-a|$.

Il criterio presenta il fianco a ~~d~~ una obiezione. Se vi è opacità completa da una parte, il calcolo della media ponderata non appare sensato.

Ma è strano invece che l'A. non si sia reso conto che ha a disposizione uno strumento per misurare separatamente la densità o la trasparenza rispetto ai due sconti e cioè da un lato $f(r-a)$ e dall'altro $f(r-q)$, strumenti molto più interessanti della sua media ponderata.

Osservazioni

1. Non si possono definire le due situazioni stimolo che danno luogo alla trasparenza in base al fatto che in una c'è una parte che sporge. Nelle situazioni più complesse non è possibile ricorrere alla "parte che sporge", che non si riesce a individuare, mentre sono sempre riconoscibili, nella trasparenza completa A e B, regioni dello sfondo, viste in parte per trasparenza, e P e Q, regioni trasparenti. È questa la definizione che comprende tutti i casi.

2. In Fig. 1A, facendo sporgere il quadrato Q in modo da ottenere B, bisogna vedere che cosa diventa trasparente

* Se restano trasparenti P e Q, si ha da un lato la valutazione di B, visto come opaco ~~e estraneo~~ e dall'altro Q, visto per trasparenza. Ad ogni modo la trasparenza di P e Q in questa situazione è in contrasto ~~non è~~ con la situazione figurale ~~ottime naturalmente~~ ed è diversa dalla forma normale di trasparenza [figura].

3. ~~Cosa c'entra la densità del filtro con il contrasto~~

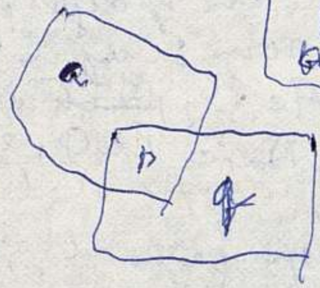
4. Se nel caso delle 3 superfici c'è meno densità nel filtro significa che nel caso standard c'è meno trasparenza; quindi nel caso standard va più colore al filtro e meno allo sfondo come prevedibile ~~dalla~~ secondo la teoria della scissione cromatica

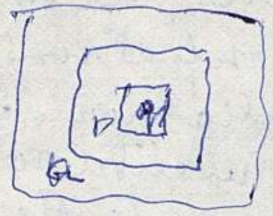
3) Probabilmente nel modello a 3 campi e anche nell'altro il rapporto funge da schermo di riduzione, cioè i colori stimolo sono colori di riduzione

* Va notato infatti che ~~...~~ figuralmente tendono a diventare trasparenti Q e B, cioè B tende ad assumere le funzioni di Q

$$\frac{p-q}{p-b}$$

$$d_{part} = \cancel{\frac{p-q}{p-b}} \quad \frac{p-q}{a-q}$$





Nella formula dell'effetto Fuchs:

cos'è a sfondo

p } regioni
q } trasparenti

Quanto alla frase conclusiva, si tratta semplicemente di una dichiarazione falsa. In primo luogo il confronto tra le due formule non è possibile perché una si applica soltanto all'effetto Fuchs e l'altra a tutti gli altri casi di trasparenza.

Sulla misura della trasparenza

Esame critico di due lavori di S. C. Masin

In un suo primo lavoro "A psychophysical study of Fuchs phenomenon" Masin studia un caso di trasparenza osservato da Fuchs (1923) in cui una figura che giace su uno sfondo omogeneo è vista per trasparenza attraverso un filtro. In questo caso, a differenza degli altri casi più studiati sono in gioco soltanto 3 superfici e secondo l'Autore non si può applicare la teoria di Metelli perché v'è un'incognita in più e quindi la coppia delle equazioni che si impostano di solito, interpretando la trasparenza come scissione fenomenica, non si può risolvere.

M. compie dei esperimenti.

1. Vengono presentati a 20 soggetti individualmente 60 diverse combinazioni di ~~filtro figura~~ sfondo, figura e filtro (2 sfondi × 5 figure × 6 filtri, tutti acromatici) con il compito di attribuire ~~a~~ in ogni combinazione un numero da 1 a 9 ad ogni filtro a seconda del grado di trasparenza percepito.

I risultati, espressi in due Tabelle e un diagramma si può così riassumere.

Il grado di trasparenza soggettiva del filtro diminuisce col diminuire della differenza di colore tra figura e sfondo. In base ai risultati di Fuchs, che afferma che il filtro risulta opaco quando questa differenza è nulla, in quest'ultimo caso la trasparenza è considerata zero.

Siccome un esperimento analogo ~~citato~~ è riportato nel Manuale di Koffka, Masin lo esamina e lo discute alla luce di una critica presentata allora da Bourdon circa l'interpretazione di Koffka. 2

In Koffka c'era un quadrato giallo su sfondo nero, il tutto coperto dal velo azzurro di un episcotista. Questa è la descrizione fenomenica: se invece si guarda il quadrato coperto dal velo dell'episcotista attraverso un filtro di riduzione, il colore di riduzione è grigio. Koffka interpretava la trasparenza come scissione cromatica del grigio di riduzione (il colore che arriva ai nostri occhi) in blu, colore dell'episcotista, e quindi giallo.

Bourdon, ripetendo l'esperimento di Koffka, sostiene che avvicinando il quadrato a pochi millimetri dall'episcotista la trasparenza è abolita [il che non si riscontra] e il colore del quadrato rimane giallastro per effetto di contrasto. Quindi il fenomeno sarebbe dovuto a contrasto.

Il secondo esperimento di Masin dovrebbe portare chiarimento su questo dibattito.

In questo esperimento (ancora 60 situazioni – 2 × 5 × 6 – ma i filtri sono diversi) ai soggetti si chiede di valutare su una scala da 1 a 9 il colore della ~~[illegible]~~ figura e dello sfondo.

Dai risultati Masin, argomentando in modo poco chiaro conclude che i soggetti hanno stimato il colore di riduzione delle figure pur vedendole attraverso il filtro. E questo darebbe ragione a Bourdon contro Koffka

In conclusione il procedimento usato da Metelli e il proced. da lui adottato per altre forme di trasparenza non può essere estraneo all'effetto Fuchs.

<u>Osservazioni</u>

1. Per quanto riguarda le critiche di Bourdon a Koffka che si basano fondamentalmente sull'osservazione di Bourdon che portando la figura a pochi millimetri dall'episcotista ~~etta~~ la trasparenza scompare. Un simile effetto non è stato mai osservato. Non solo, ma il fatto che si vede la trasparenza anche con mosaici opachi dimostra che non è affatto necessario che ci sia una effettiva distanza fra ciò che appare il velo trasparente e la figura vista per trasparenza. Inoltre l'effetto di contrasto dovrebbe prodursi anche <u>dietro</u> il velo trasparente e a notevole distanza (su un altro piano), il che non corrisponde ai fatti.

2. Per quanto riguarda il fatto che il colore giudicato della figura dietro il filtro sarebbe <u>pari</u> al colore di riduzione, non c'è niente da obbiettare. I soggetti in seguito al compito di giudicare il colore della piccola figura al centro del campo, la astraggono dal resto e realizzano la percezione in riduzione. Il che non vuol dire affatto che, assumendo un atteggiamento meno spiccatamente analitico, debbano continuare a vedere il colore di riduzione.

In un successivo lavoro "An experimental comparison of three- versus four-surface phenomenal transparency" Metelli affronta il problema della diversità dei due tipi di trasparenza considerati nel precedente articolo, cercando una teoria che li accomuni. Nel far questo egli considera rappresentativo del tipo più comune di trasparenza, il caso in cui la figura che nel fenomeno di Fuchs è dietro il filtro viene spostata in modo da sporgere dalla parte dello sfondo ~~vista~~ che per i soggetti non è ricoperta dal filtro.

1° Esperimento

33 soggetti dovevano giudicare la densità del filtro in 45 situazioni (5 filtri per 9 ~~quasi~~ figure sullo stesso sfondo) due volte, una relativamente alla situazione a 3 superfici e l'altra, essendo spostata la figura, in una situazione a 4 superfici.

Risultati

C'è differenza fra i risultati nei due tipi di trasparenza, ma la differenza può essere spiegata dato che fin da T.H. è stato osservato che la nettezza dei contorni di un oggetto diminuisce l'impressione di densità, in questo caso la differenza fra p e q (regioni dell'ambito della trasparenza) viene aumentata per contrasto creato dalla parte prominente b. Quindi la presenza della regione B non avrebbe alcun effetto funzionale.

Le curve piatte di 3 soggetti si spiegherebbero col fatto che questi tre soggetti valutavano soltanto la densità del filtro sullo sfondo e tale densità (b-a) è pressoché uguale variando le riflettanze delle figure. M. ritiene che ci debba essere anche (1) un gruppo opposto di caratteristiche, influenzati soltanto principalmente dalla differenza p-q e quindi non osservavano la trasparenza funzionale. Questo formerebbe un sottogruppo di 19, mentre un sottogruppo di 44 riportano la trasparenza non-funzionale.

A conclusione delle predette osservazioni stato tentato di ricavare una formula che misuri la densità del filtro tenendo conto di quelle che secondo lui sono le variabili che determinano la percezione della trasparenza e cioè la differenza in colore di riduzione tra filtro e sfondo (p-a) e la differenza in colore di riduzione fra figura e filtro (p-q). Le parti che sporgono sotto il filtro non influirebbero funzionalmente sul meccanismo di formazione della trasparenza parziale, ma solo indirettamente attraverso al contrasto.

Nel costruire un modello che si adatti a questi risultati, M. sceglie una funzione di potenza, perché queste funzioni descrivono una quantità di dati sensoriali.

La densità del filtro espressa dalla differenza (p-a) sarebbe espressa da $\Psi_{p,a} = K(p-a)^{B}$, mentre l'effetto della differenza p-q sarebbe espresso da $\Psi_{p,q} = K(p-q)^{B}$. La grandezza Ψ dell'impressione generale di densità sarebbe espressa da una media ponderata di $\Psi_{p,a}$ e $\Psi_{p,q}$; media ponderata perché - - -

La formula è $\Psi = K\left[W(p-a)^{B} + (1-W)(p-q)^{B}\right]$

in cui W è un coefficiente di peso individuale. Nei tre soggetti W=1; per il gruppo degli 19 W è vicino a 0, per gli altri sta fra 0 e 1.

Gli altri due esperimenti sono stati fatti per vedere se la formula si può semplificare.

Esper. 2. I soggetti vengono sottoposti a 48 situazioni (16 sfondi × 3 filtri). In una prima presentazione dovevano rispondere (valutare) che densità aveva il filtro davanti alla figura, nella 2ª (1ª per altri soggetti) che densità aveva il filtro davanti allo sfondo. I risultati nelle due situazioni sono diversi, quindi i soggetti hanno corrisposto alle diverse istruzioni. Il coefficiente w non si può equiparare a 1

~~Esp. 3~~

Esp. 3 Se tenendo costante p e a, t varia linearmente con q, ~~il coefficiente β~~ (il coefficiente $\beta = 1$) $= 1$.
Sono stati usate 24 situazioni (3 filtri × 8 figure)

Ai soggetti è stato chiesto prima di valutare la densità apparente del filtro in ogni situazione e poi, in diverso ordine, la bianchezza della figura vista attraverso il filtro (Sec. Metelli in questi casi i soggetti valutavano la bianchezza di zone). Come risultato si è ottenuta una relazione lineare delle valutazioni. Quindi la formula si semplifica essendo $\beta = 1$

13)

Osservazioni (2)

1. Nel suo secondo lavoro (An exp. comparison) Masin mette a confronto il fenomeno di Fuchs con un particolare caso di trasparenza che si ottiene spostando la figura in modo che sporga dal limite tra filtro e sfondo, ottenendo in tal modo una situazione a 4 superfici. La trasparenza risulta leggermente diversa, ma Masin attribuisce tale diversità al contrasto prodotto dalla parte protrudente e ~~in seguito~~ a tale risultato considera insieme le due situazioni, considerando senza importanza la parte protrudente, ed attribuisce carattere fondamentale all'effetto Fuchs, partendo da questo per costruire una formula di misura dell'opacità (che praticamente corrisponde ad una misura della trasparenza).

A questo punto conviene tener presente che il caso eccezionale, in quanto per verificarsi richiede anche la disparità binoculare, è l'effetto Fuchs, mentre il caso ottenuto con la protrusione rappresenta una variante di un gran numero di situazioni di trasparenza a 4 campi. Conviene quindi, se mai, seguire la via opposta e considerare l'effetto Fuchs come una variante per difetto del caso più generale, e se è vero che la differenza di trasparenza fra i due casi è trascurabile, usare l'espediente di far sporgere la figura dal filtro per unificare il caso Fuchs agli altri, rendendo in tal modo applicabili le note formule.

Masin procede invece in modo opposto, considera l'effetto Fuchs come punto di partenza per ricavare una formula valida anche per i comuni effetti di trasparenza. La formula non comprende la parte B e quindi trascura una caratteristica rilevante, una delle 4 superfici, dei casi a 4 superfici.

Sembra che data la scarsa rilevanza di B nel caso particolare da lui considerato, la sua formula debba essere utilizzata senza modificazioni per la misura della trasparenza anche nelle altre situazioni.

2. Vediamo ora di analizzare l'equazione di Masin, mettendola a confronto con l'equazione di Metelli: 8

$$\psi = K\left[w(p-a) + (1-w)(p-q)\right] \quad \text{(Masin)}$$

in cui ψ è un indice di opacità

$$\alpha = \frac{p-q}{a-b} \quad \text{(~~indice~~ Metelli)}$$

in cui α è un ~~coefficiente~~ indice di trasparenza

L'indice α è stato sottoposto a una serie di controlli; sarà utile vedere come risponde ψ agli stessi controlli.

a) 1. Se $p = q$ $\alpha = 0$ cioè la trasparenza è nulla, il che corrisponde ai fatti

2 Se $p = q$

$\psi = K[w(p-a)]$. Cioè ψ anziché diventare massimo, diminuisce per la perdita di un termine. ~~Manca da~~ In altre parole la densità del filtro diminuisce anziché aumentare

b) Se (1) fermi restando a e b, la differenza fra p e q diminuisce secondo l'equaz. $\alpha = \frac{p-q}{a-b}$ diminuisce la trasparenza (cioè aumenta l'opacità), ciò che corrisponde al fatto

Se la differenza fra p e q diminuisce,

(2) nella formula di Masin diminuisce il 2° termine cioè la misura dell'opacità anziché aumentare diminuisce

c) la formula proposta da Masin non ha né un massimo (opacità). ~~[illegible]~~ K corrisponde alla né un minimo trasparenza assoluta?. ~~Il minimo non è zero~~ perché ~~la formula pur~~ anziché all'opacità ~~l'indice può essere negativo. È un significato particolare~~ [illegible] ~~negativo~~? 15)

$$t = \frac{aq - pb}{(a+q)-(b+p)}$$

$$= \frac{aq - pb}{(a-p)+(q-b)}$$

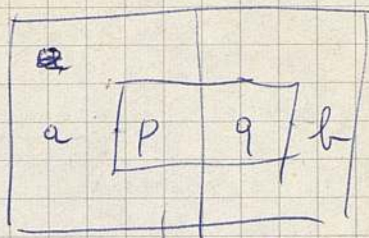
$$= \frac{aq - pb + pq - pq}{2(a-p)}$$

$a-p$

$$= \frac{q(a-p) + p(q-b)}{2(a-p)}$$

$$= \frac{\cancel{(a-p)}\,(q+p)}{2\,\cancel{(a-p)}}$$

$$= \frac{p+q}{2}$$



10

90 70 30

$a > p > q > b$

$\boxed{a - p = q - b}$

$90 - 70 = 30 - 10$

$20 = 20$

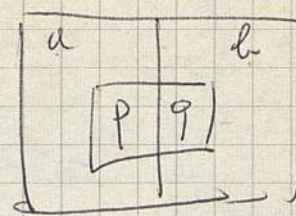
$a > p > b > q$

$a > b > p > q$

$p > q > a > b$

$p > \cdot > q > b$

S. preparzonul
astical
Matisse

$$t = vp + (1-\cancel{p}v)\,q$$ 

$$v = \frac{q-b}{(a-p)+(q-b)}$$

$$t = \frac{aq - pb}{a-p+q-b} = \frac{aq - pb + pq - pq}{a-p+q-b} =$$

$$= \frac{q(a-p) + p(q-b)}{(a-p)+(q-b)} =$$

$$= \frac{q(a-p)}{(a-p)+(q-b)} + \frac{p(q-b)}{(a-p)+(q-b)}$$

$$\underbrace{\frac{(a-p)}{(a-p)+(q-b)}}_{v} + \underbrace{\frac{(q-b)}{(a-p)+(q-b)}}_{1-v} = 1$$

$$v = \frac{a-p}{(a-p)+(q-b)} \qquad 1-v = \frac{q-b}{(a-p)+(q-b)}$$

$$t = q\cdot v + p(1-v)$$

$$t = \frac{aq - pb}{(a - p + \cancel{b}\,q - \cancel{q}\,b)}$$

$a - b - p + q$

$a - p = q - b$

$\quad\quad\ \ b - q$

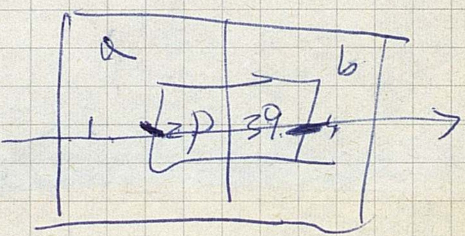
$p - q = q - b$

$\quad\quad\ \ b - q$

$$t = \frac{aq - pb}{(a+b) - (p+q)}$$

$a - p = q - b$

$$= \frac{aq - pb}{a - p + b - q}.$$



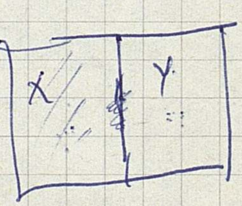
$$|a - p| = \cancel{q - b}\,|b - q|$$

$|a - p| = |b - q|$

~~$a > p > q > b$~~

~~$a \neq p = q - b$~~ ~~$a > p > q > b$~~

$$t = \frac{p+q}{2}$$

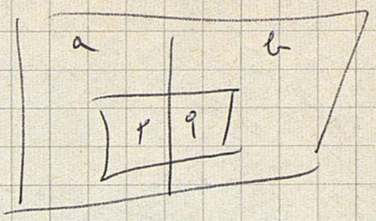


$\downarrow$

$-(x+y)$

$+1$

$$t = \frac{aq - pb}{\quad}$$



$$t = \frac{aq - pb}{a - p + q - b}$$

$$= \frac{aq - pb + ab - ab}{a - p + q - b}$$

$$= \frac{a(q-b) + b(a-p)}{(a-p) + (q-b)}$$

$$= \frac{a(q-b)}{(a-p)+(q-b)} + \frac{b(a-p)}{(a-p)+(q-b)}$$

$$\underbrace{\frac{q-b}{a-p+q-b}}_{1-u} + \underbrace{\frac{a-p}{a-p+q-b}}_{u} = 1$$

$$\begin{cases} t = a(1-u) + bu \\ t = q \cdot u + p \cdot (1-u) \end{cases}$$

$\alpha = \alpha'$

$t = t'$

$= .8$ [illegible] $.1$ [illegible]

$= .6$ [illegible] $.3$ [illegible]

$$0 = a(1-u) + bu - qu - p(1-u)$$

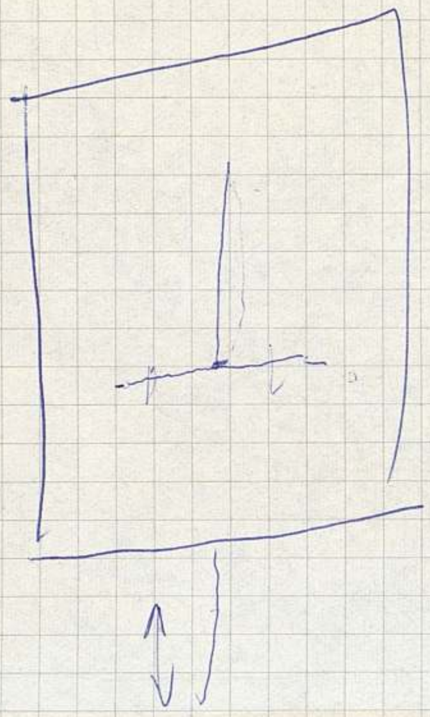
$$0 = a - au + bu - qu - p + pu$$

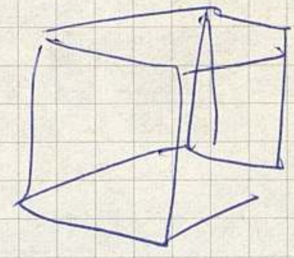
$$0 = a - p + u(b - a - q + p)$$

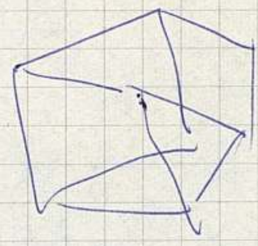
$$0 = (a - p) - u(-b + a + q - p)$$

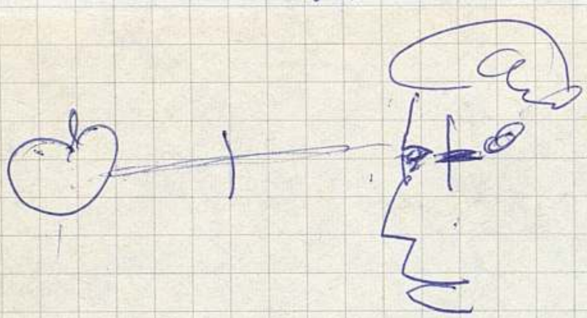
$$u(a - p + q - b) = a - p \qquad u = (a-p)/(a-p+q-b)$$

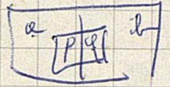
18











$$t = \frac{aq - pb}{a - p + b - q} \qquad ② = a-p = q-b \qquad ① = a-p = b-q$$

se $a-p = b-q$

① $$t = \frac{aq - pb}{2(a-p)} = \frac{aq - pb + pq - pq}{2(a-p)}$$

$$= \frac{q(a-p) + p(q-b)}{2(a-p)}$$

$$= \frac{q\cancel{(a-p)} - p\overset{(a-p)}{\cancel{(b-q)}}}{2\cancel{(a-p)}}$$

$$= \frac{\cancel{(a-p)}(q-p)}{2\cancel{(a-p)}} = \frac{q-p}{2}$$

② $$t = \frac{p+q}{2}$$

$$t = vp + (1-v)q$$

$$v = \frac{b-q}{(a-p)+(b-q)}$$

$(a+q) - (b+p)$

$a + \cancel{q} - b - \cancel{p}$

$$t = \frac{b-q}{(a-p)+(b-q)}\,p + 1 - \frac{b-q}{(a-p)+b-q}\,q$$

$$t = \frac{pb - pq}{a-p+b-q} + \frac{a-p+b\cancel{-a}\cancel{-b}+\cancel{q}}{a-p+b-q}\,q$$

$$\cancel{t = \frac{pb - pq + aq - bp + bq - q^2 - bq + q^2}{a-p+b-q}}$$

$$t = \frac{pb - pq + aq - pq}{a-p+b-q}$$

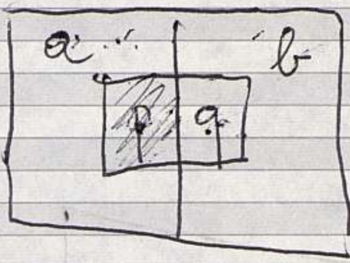
$a + p - b - q$

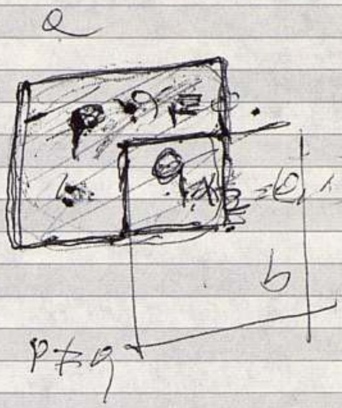
$$t = \frac{pb - pq \cancel{\ldots} - a + p - b - q}{}$$

$pb - \cancel{pq}^{a} - aq + \cancel{qq} \cancel{\ldots} bq - bq - q^2$

$$t = \frac{pb - qp}{a+p-b-q} + \frac{aq + \cancel{bq} - pq\,\cancel{q^2} - \cancel{bq} + \cancel{q}^2}{a+\cancel{p}-\cancel{b}-q}$$

$bp - \cancel{aq} \; \cancel{aq} + pq - \cancel{b}q - q^2 + \cancel{b}q - q^2$





$$p = \alpha_1 a + (1-\alpha_1) t$$

$$q = \alpha_2 b + (1-\alpha_2) t \qquad \alpha_p = \frac{q-p}{b-p}$$

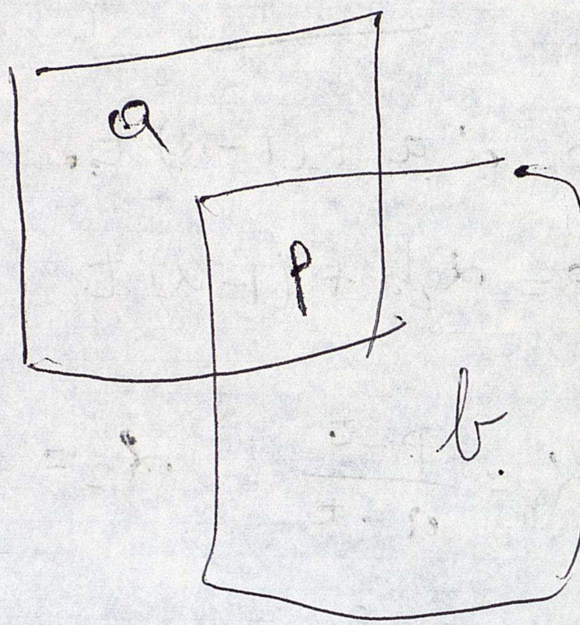
$$\alpha_1 = \frac{p-t}{a-t} \qquad \alpha_2 = \frac{q-t}{b-t}$$

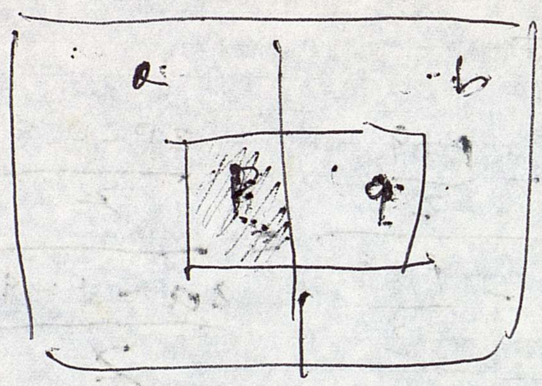
$$t = \frac{p+q}{2}$$

$$t = \sqrt{pq}$$

$$\alpha_1 = \frac{p - \frac{p+q}{2}}{a - \frac{p+q}{2}} = \frac{\frac{2p-p-q}{2}}{\frac{2a-p-q}{2}} = \frac{p-q}{2a-p-q}$$

$$\alpha_1^* = \frac{p-\sqrt{pq}}{a-\sqrt{pq}}$$





$\alpha_1 = 0$. $a - p = k$

$\alpha_1 = 0 \qquad p = q$

$\alpha_1 = 1 \qquad p - \cancel{q} = 2a - p - \cancel{q}$

$$2p = 2a$$

$$a = p$$

$\alpha_1^* = 0 \qquad p = \sqrt{pq}$

$$p^{\cancel{2}} = \cancel{p}q$$

$$p = q$$

$\alpha_1^* = 1 \qquad p - \cancel{\sqrt{pq}} = a - \cancel{\sqrt{pq}}$

$$p = a$$

$p = 9 \qquad \sqrt{pq} = 6$

$q = 4.$

$$\frac{p+q}{2} = \cancel{6}\ \frac{13}{2} = 6.5$$